Chronology of SGA 5

Luc Illusie

Grothendieck's seminar *Cohomologie ℓ-adique et fonctions L* [13], later[1] labeled SGA 5, extended itself over two periods: October 1964 – June 1965, January 1966 – June 1966.

October 1964 – June 1965

In his first exposés, Grothendieck reviewed the formalism of derived categories, the basic theorems in étale cohomology (proper base change, smooth base change, ...) and the global duality theorem, which had been established in SGA 4, following the lines of the sketch given by Verdier in [23]. The proof presented in exposés XVII and XVIII of the published version of SGA 4 [2], which is based on a different approach to the functor $f^!$ and the trace morphism, is due to Deligne and was written up by him several years later.

Then Grothendieck moved on to a part that he considered to make the real beginning of the seminar, namely, local duality. He introduced the notion of dualizing complexes, discussed their uniqueness and basic properties, formulated, for the first time, the *conjecture of absolute purity*, and proved that modulo this conjecture plus resolution of singularities, on a good regular scheme X the constant sheaf $\mathbb{Z}/n\mathbb{Z}$, for $n \in \mathbb{Z}$ invertible on X, is dualizing. It is to be noted that Grothendieck didn't use the word "operation" to denote a functor, in particular, didn't give any talk on the "formalism of six operations".[2] But he proved various remarkable formulas concerning the interaction of these functors, such as the fact that the dualizing functor[3] exchanges ! and $*$.

The next topic was the Lefschetz-Verdier trace formula. He discussed cohomological correspondences, and explained the construction of the so-called *Verdier pairing*, and the definition of the *Verdier local terms*, which, except in low dimensional cases, were conditional on the validity of the resolution of singularities. He then stated the Lefschetz-Verdier trace formula, i.e., the compatibility of the formation of these local terms with proper maps, but did not prove it. He simply said that checking the required compatibilities was a routine exercise, which should probably be rather long. He didn't discuss its application to transversal endomorphisms of curves, due to Verdier, for

[1]During the course of the oral seminar, the previous seminars had no numbering: SGA 3 was referred to as SGAD (D for Demazure), and SGA 4 as SGAA (A for Artin). Grothendieck chose the numbering SGA 1, etc., when SGA 6 started.

[2]The first appearance of the functor $f^!$ was in the coherent sheaves context, in the notes he wrote in the summer of 1963 [15].

[3]Now usually called *Verdier dual*, though its construction is due to Grothendieck.

which the hypotheses are satisfied, but requires additional arguments [24]. The reason is that he had a full proof of this application, independent of the Lefschetz-Verdier formula and free of any resolution assumption, based on the theory of Nielsen-Wecken traces, that he would explain at length later in the seminar. At the end, he suggested the possibility of a variant of the Lefschetz-Verdier formula in the context of coherent sheaves, leading to a generalization of the so-called *Woods Hole formula* (([6], p. 150), [3]), but he didn't elaborate.

Grothendieck then gave a series of exposés on the construction of cycle classes, and their compatibility with intersection and Gysin maps. In particular, he introduced, for the first time, the notion of *homology* of a complex as cohomology (with a change of signs in the degrees) with values in its dual, in the sense of the dualizing functor applied to it[4].

The next talks were given by Jean-Pierre Jouanolou on his ongoing work, which was to be part of his thesis. The largest part of them was devoted to the definition of (constructible) $\mathbb{Z}_\ell$- and $\mathbb{Q}_\ell$-sheaves and the construction of certain cohomological functors on them, such as $R^i f_!$, which, at the end of the second part of the seminar, would enable the formulation and proof of Grothendieck's theorem of rationality of L-functions. A derived category formalism for ℓ-adic complexes was to be defined much later, by Deligne and others. His last talks were on an independent topic, namely the calculation of the étale cohomology (for torsion coefficients) of certain classical schemes, such as projective bundles, the construction of Chern classes, and the proof of the so-called self-intersection formula in the Chow ring, a formula due to Mumford, and its application to the calculation of étale cohomology of certain blow-ups.

January 1966 – June 1966.

This part started by two exposés of Grothendieck (Jan. 4 and 7, 1966)[5] where he briefly recalled the Lefschetz-Verdier formula, and its application to the case of curves and transversal endomorphisms (which he said would be treated by another method later in the seminar), and moved on to Euler-Poincaré characteristics of schemes with finite group actions, announcing the Grothendieck-Ogg-Shafarevitch formula, and discussing variants and conjectures for analytic or topological spaces.

Then Grothendieck proceeded to the proofs of the two major theorems of

[4]Later, this construction was referred to as *Borel-Moore homology*, though no dualizing complex appears in the original article of Borel-Moore [5].

[5]Called *exposé introductif* in the introduction of [13].

the seminar, namely:

(i) the Grothendieck-Ogg-Shafarevitch formula, that Raynaud presented in 1966 at the Bourbaki seminar [20];

(ii) a Lefschetz trace formula on curves.

The local terms of (i) involve Swan conductors. Serre gave lectures on the Swan module, published independently [21]. Grothendieck had presented (ii) in 1966 at the Bourbaki seminar [11] shortly before Raynaud, using the method of the Lefschetz-Verdier trace formula. Not waiting for Verdier to check the compatibilities of his formula and write up the details of the application to curves, Grothendieck gave his own proof, alluded to above. For this he developed a formalism of non-commutative traces generalizing that of Stallings [22], and by a method inspired by the (much older) work of Nielsen and Wecken proved the desired Lefschetz trace formula on curves.

The last part consisted of exposés by Christian Houzel. After preliminaries on the Frobenius correspondence in étale cohomology, he used the formalism of ℓ-adic cohomology previously constructed by Jouanolou to define the L-functions of ℓ-adic sheaves on schemes over finite fields, proved their main formal properties, and eventually deduced from the trace formula for curves Grothendieck's cohomological expression for L-functions, which was the culminating point of the whole seminar. Grothendieck must have given the last talk but, unfortunately, I have no memory nor any document about its date and its contents.

The writing up

Exposés I, II, III

I wrote up I and III in the first semester of 1966. For this, I used the handwritten notes I had taken. Grothendieck didn't give me any personal notes. He made many comments on my first drafts, that we discussed at length at his place. He was satisfied with the final versions. For I, this is the version in [13]. Both I and III were faithful transcriptions of Grothendieck's talks. In particular, the Lefschetz-Verdier local terms were defined modulo resolution assumptions, the formula itself was admitted, and no application to transversal endomorphisms of curves was given.

At the same time I also wrote up notes that Grothendieck handed me on Künneth formulas, generic cohomological properness and local acyclicity. They didn't correspond to any oral exposé, and Grothendieck labeled them II. The main statements were conditional on resolution of singularities. Again, he was satisfied with the final drafts. These versions of I, II, III were typed by the IHES, mimeographed, and distributed the same year.

I will explain further below the story of the publication of II and III.

Exposé IV

Grothendieck asked Jouanolou to write up his exposés on the cycle class and homology. Jouanolou made a preliminary draft (Exposé IV), of which Grothendieck was not satisfied. A full revision was needed, and Grothendieck told me that he was afraid of having to do it himself.[6] One serious obstacle to an immediate re-writing was that the construction of cycle classes heavily depended on the global duality theory of SGA 4, namely, the properties of the functors $f^!$ and $f_!$, and the trace map. Grothendieck had asked Deligne to write it up. Deligne used the Verdier approach[7] that he had just successfully applied in his appendix to Hartshorne's seminar [16]. Because he wanted to write solid foundations on the formalism of derived categories, especially on the question of signs, and that on his way he was discovering new results,[8] the writing took him much longer than expected. He had also to use at certain places his theory of cohomological descent, which was written up by Bernard Saint-Donat in ([2], Vbis) and was not immediately available. Grothendieck wrote the introduction to [2] in November, 1969. At that time he was interested in other mathematical topics (crystalline cohomology and Dieudonné theory), and was gradually absorbed by new political preoccupations. The revision of IV was never made.

Exposés V, VI, VII

Jouanolou wrote up his exposés on the ℓ-adic formalism and Chern classes. He finished by 1970.

Exposé XIV

Houzel wrote up his exposé in 1966. Grothendieck was satisfied, and the mimeographed text was then distributed by the IHES.

Exposés VIII, X, XI, XII

Ionel Bucur was in charge of writing up Grothendieck's exposés on the Grothendieck-Ogg-Shafarevich formula and the Lefschetz trace formula on curves. Except for a couple of short visits to France he was in Romania, working in very difficult conditions, and his writing was unfortunately not finished until 1972.

I have no information on the precise date at which the writing of Exposés VIII and X was finished, but it must have been before 1972.

---

[6] This happened from time to time. For example, Grothendieck was not happy with Verdier's first draft of SGA 4, Exp. IV [2], that he eventually totally re-wrote (with Verdier's collaboration).

[7] Definition of $f^!$ as a right adjoint to $f_!$.

[8] Such as the symmetric Künneth formula ([2], XVII Th. 5.5.21). See the introduction of [2] for a list of them.

In Dec. 1972, Deligne, who was at Harvard, received Bucur's write-up of Exposé XII and sent it to the IHES to be typed. On Feb. 4, 1973, Bucur wrote me that he was concerned about the draft of his Exposé XI, that he had seen for the last time in Grothendieck's room at IHES, and had not received any news from him about it. It seems that his text was lost when Grothendieck moved from the IHES. Unfortunately, Bucur had no copy of it. On Jan. 28, 1974 Bucur wrote me that he was still thinking about the local terms of the trace formula. I wrote him back asking him to tell me more about this, and informing him that his Exposé XII had been distributed by the IHES, but that his Exposé XI had probably been lost in Grothendieck's moving. Bucur was already ill, and our correspondence stopped. He died on Sept. 6, 1976.

The introductory and closing exposés

The introductory exposé consisted of the two talks given by Grothendieck at the beginning of 1965, that I have mentioned above. Grothendieck had not assigned the writing up of these talks to any participant of the seminar, and had not distributed any personal notes. It was tacitly assumed that he would write them up himself. He did so for the introductory and closing exposés of SGA 6 [4].

Finalization?

In 1974 the question was whether the existing write-ups of the exposés could be assembled into a volume.

A critical point was that the mere statement of the Lefschetz formula needed for proving Grothendieck's trace formula for Frobenius and the cohomological interpretation of L-functions in Exposé XIV could not be found in the existing write-up of XII.[9] It might have been possible to deduce it from the contents of XII (as probably Bucur was trying to do in 1974), but the proof would have been incomplete, as XII relied on the formalism of the lost exposé XI. Even if XI had been recovered, XI and XII needed to be carefully revised by Bucur in close coordination with Grothendieck. That would not have been possible, as at the time Grothendieck was campaigning for stopping mathematical research and had other occupations and interests. On the other hand, as explained above, the Lefschetz-Verdier formula of III had not been checked and its application to curves not given, hence was of no help.

Also, the absence of Exposé IV (not to mention that of the introductory and closing exposés) posed problem.

[9] For the local terms to have the simple form as the trace of the endomorphism on the stalks of the sheaf at the fixed points, transversality of the endomorphism of the curve with respect to the diagonal is essential, and this was nowhere discussed in Exposé XII.

What to do?

Two events

In 1973 - 1974 two unrelated events happened, which were to have a crucial impact on the edition of the seminar.

(a) In June 1973, Deligne announced he had proven the Weil conjecture about the eigenvalues of Frobenius on ℓ-adic cohomology of projective, smooth varieties over finite fields. He explained his proof in six talks at a conference held in July, 1973, in Cambridge in honor of Hodge, and quickly wrote it up. It was published in [7]. The proof relied on Grothendieck's Lefschetz formula recalled in ([7], (1.5.1)). Concern started growing on the fact that no written account of the proof of this formula was available.

(b) In 1973-74 Deligne was mostly working on a generalization of [7], which was to become *Weil II* [9]. But, quite unrelated to this, on Jan. 7, 1974, he wrote a letter to Mike Artin, in which he proved unconditionally the stability of constructibility by direct images for morphisms of finite type over a field, and sketched important complements in generic situations, and similar finiteness theorems for nearby cycles and dualizing complexes. Soon afterwards, he wrote up the details in what was to become ([8], Théorèmes de finitude).

The genesis of SGA 4 1/2

The results in (b) made it possible to re-write Exposés II and III without hypotheses of resolution, and desirable to check the compatibilities needed for the proof of the Lefschetz-Verdier formula. On May 20, 1974, Deligne wrote me a letter suggesting such a re-writing of II, using the contents of his letter to Artin, and giving a proof of a conjecture Grothendieck had made in II, using the notion of cospecialization map. I didn't work on it until Oct. 1976.

On May 28, 1974, Deligne wrote me again, about III this time, sketching a strategy for the verification of the Lefschetz-Verdier formula. I worked about this during the winter of 1974-75, and I completed the verification by the spring of 1975. He proposed that as an application I wrote a proof of a statement Langlands had made in ([18], Proposition 7.12) (without proof). This statement was a far reaching generalization of Verdier's formula ([24], 4.1). And it contained, as a special case, Grothendieck's trace formula. It was unclear how to prove Langlands' statement by Grothendieck's Nielsen-Wecken method, but it looked feasible to apply the (now established) Lefschetz-Verdier formula to deduce it by a suitable adaptation of Verdier's arguments in [24]. In the summer of 1975, I succeeded in doing this, and, at the same time, I showed the coincidence of Lefschetz-Verdier local terms with those defined by Grothendieck by means of the Nielsen-Wecken method, developing for this a sheafified version of the theory of non-commutative traces of the (missing) XI.

A Summer Institute in Algebraic Geometry, organized by the AMS, had been held at Arcata, California, in July and August, 1974. An important part of it was a seminar, chaired by Artin, on Deligne's proof of the Weil conjectures and of the Hard Lefschetz theorem (which was to be part of [9]). As a preparation, Deligne gave 7 lectures on the basics of étale cohomology. However, they didn't include the formalism of ℓ-adic cohomology, that he had developed in the context of derived categories in [7] (and superseded that of Jouanolou), nor Grothendieck's trace formula.

In the fall of 1974, Deligne had no idea how long it would take me to check the Lefschetz-Verdier formula and give the required application to Grothendieck's trace formula, nor even if I would eventually succeed. It was becoming more and more urgent to make a proof of it available. That's why he decided to quickly write up a self-contained, neat proof of Grothendieck's trace formula for Frobenius, independent of Bucur's write-up of XI and XII, with the simplifications brought by the use of the notion of *perfect complex*, which was not available at the time of the oral seminar.[10] In fact, more was needed, namely the notion of filtered derived category, and the corresponding additivity of traces ([8], Rapport, (4.4.1)).[11]

In the course of this writing, Deligne realized that he could prove (and he quickly wrote it up) a souped up version ([8], fonctions L modulo ℓ^n et modulo p, Th. 2.2) of the trace formula of ([8], Rapport, 4.10), for torsion coefficients. The key new ingredient was the symmetric Künneth formula he had established in ([2], XVII 5.5).

Deligne was still concerned with the absence of Exposé IV. He therefore decided to do what he had done for the trace formula (and, for nearby cycles, in SGA 7 ([12], Exp. I)), i.e., quickly write up a self-contained account of the main points of Grothendieck's construction. He probably used his own notes and the memories he had of Grothendieck's talks that he attended in the first semester of 1965, but mostly reconstructed the theory by himself, with the help of the duality formalism he had developed in ([2], XVII, XVIII). However, he didn't prove the compatibility of cycle classes with Gysin maps, nor did he discuss the formalism of homology constructed by Grothendieck.

[10]It was to be developed in SGA 6 [4] and became standard afterwards.

[11]Daniel Ferrand discovered in 1968 that, in general, traces are not additive on endomorphisms of perfect complexes [10]. Soon afterwards, a satisfactory formalism (filtered derived categories), where additivity was restored was constructed in ([17], V). However, this (wrong) addivity is implicity used in Bucur's XII, (5.3), referring to the (missing) XI, 4. This should have been fixed in the expected revision.

In 1974-75, A. Douady and J.-L. Verdier ran a seminar at the ENS around the Baum-Fulton-Mac Pherson's version of the Riemann-Roch theorem and various questions in étale or singular cohomology. Bernard Angéniol gave a talk on Deligne's finiteness theorems [1], Verdier gave talks on constructibility and homology in topological or complex analytic set-ups [25], and Gérard Laumon on the construction of homology classes in étale cohomology, parallel to Deligne's write-up, but using Grothendieck's homology formalism[12] and proving the compatibility with Gysin maps.

It is probably in the course of 1975 that Deligne conceived the idea of assembling Jean-François Boutot's notes on his exposés at Arcata plus the various pieces he had just written up (proof of the trace formula and of its mod ℓ^n and mod p variants, finiteness theorems, cycle class, plus complements to global duality[13]) into a separate publication. In his spirit it was related both to SGA 4, as Boutot's notes were a gentle introduction to étale cohomology, and to SGA 5 by the trace formula. That led him to choose the title SGA 4 1/2.

The final steps

In 1975-76 Deligne had obtained beautiful applications of Grothendieck's trace formula and of his work "Weil II" [9] (which was still in preparation) to estimates of exponential sums. He decided to include them in the future SGA 4 1/2. Verdier's thesis on derived categories and derived functors had not been published. The summary he had written up in 1963 had been superseded by other expositions (the beginning of [16] and the first part of ([2] XVII)). However, Deligne thought that it was still interesting, and that it was a good idea to include it as well, which he did with the permission of Verdier. On Sept. 20, 1976, Deligne wrote the introduction to SGA 4 1/2. In Oct. 1976, thinking again about SGA 5 II, he invited me to write up the (unconditional) results on cohomological properness and local acyclicity he had sketched in his letter to me of May 20, 1974, as they would constitute a natural complement to his write-up of his finiteness theorems in SGA 4 1/2. I did it quickly, and in Dec. 1976, he submitted the volume[14] to the Springer Lecture Notes. He also told me that in his letter to A. Dold, he had said that SGA 5 should be ready by March, 1977. That left little time.

I hurried to return to the writing up of the results I had obtained in 1974-75, namely:

[12] Laumon told me that, at the time, he was unaware that this formalism was due to Grothendieck, and he was not instructed to give proper credit for what he was reporting on.

[13] Including a crucial compatibility that had been admitted in ([2], XVIII, 3.1.10.3).

[14] This last text was included as an appendix to Théorèmes de finitude.

(i) the checking of the compatibilities in the Lefschetz-Verdier formula;

(ii) at the suggestion of Deligne, the same verification for the generalized Woods-Hole formula mentioned above ([6], p. 150);

(iii) the proof of the Langlands formula ([18] Proposition 7.12);

(iv) the sheafified version of non-commutative traces and the coincidence of Lefschetz-Verdier and Grothendieck Nielsen-Wecken local terms.

I put (iii) and (iv) together in a package that I called III B.

The manuscript was ready by February 1977.

Because of the original work I had done on the new version of III, Deligne proposed to me to be the editor of SGA 5, which I accepted. I wrote the introduction[15] on Feb. 19, 1977. I sent a copy of the whole volume to Grothendieck, asking for his observations. In a letter dated March 17, 1977, he answered: "Tout semble parfait." ("Everything looks perfect."). I then made the submission.

Acknowledgements

I am grateful to Pierre Deligne, Hélène Esnault, Michael Rapoport and Weizhe Zheng for helping me fix typos and English language inaccuracies in a first version of this text, and to Claire Voisin for suggesting improvements in the presentation.

References

[1] Angéniol, Bernard. *Théorème de finitude en cohomologie étale (d'après P. Deligne).* Séminaire de géométrie analytique (École Norm. Sup., Paris, 1974–75), pp. 152–162, Astérisque, No. 36-37, Soc. Math. France, Paris, 1976.

[2] Artin, Michael; Grothendieck, Alexander; Verdier, Jean-Louis. *Théorie des Topos et Cohomologie Étale des Schémas.* Séminaire de Géométrie Algébrique du Bois-Marie 1963-64 (SGA 4), I, II, III. Lecture Notes in Mathematics 269, 270, 305, Springer-Verlag, 1971, 1972, 1973.

[3] Atiyah, M. F. *A Lefschetz fixed point formula for holomorphic mappings.* Contemporary Problems in Theory Anal. Functions (Internat. Conf., Erevan, 1965) (Russian), pp. 28–32, Izdat. "Nauka", Moscow, 1966.

[15] p. v, l. -9, "VII" should be replaced by "VIII". p. vi, l. 2: the "déménagement" was that of Grothendieck, as explained above. p. vi, l. 3, "commutative" should be replaced by "non-commutative".

[4] Berthelot, Pierre; Grothendieck, Alexander; Illusie, Luc. *Théorie des intersections et théorème de Riemann-Roch.* Séminaire de Géométrie Algébrique du Bois-Marie 1966-67 (SGA 6). Lecture Notes in Mathematics 225, Springer-Verlag, 1971.

[5] Borel, A.; Moore, J. C. *Homology theory for locally compact spaces.* Michigan Math. J. 7 (1960), 137–159.

[6] Colmez, Pierre; Serre, Jean-Pierre (eds.). *Correspondance Grothendieck-Serre.* Documents mathématiques 2, SMF 2001.

[7] Deligne, Pierre. *La conjecture de Weil. I.* Inst. Hautes Études Sci. Publ. Math. No. 43 (1974), 273–307.

[8] Deligne, Pierre. *Cohomologie étale.* Séminaire de Géométrie Algébrique du Bois-Marie SGA 4 1/2. Lecture Notes in Math., 569, Springer, Berlin, 1977.

[9] Deligne, Pierre. *La conjecture de Weil. II.* Inst. Hautes Études Sci. Publ. Math. No. 52 (1980), 137–252.

[10] Ferrand, Daniel. *On the non additivity of the trace in derived categories.* arXiv:math/0506589v1, 2005.

[11] Grothendieck, Alexander. *Formule de Lefschetz et rationalité des fonctions L.* Séminaire Bourbaki, Vol. 9, Exp. No. 279, 41–55 (1966), in Dix exposés sur la cohomologie des schémas, 31–45, Adv. Stud. Pure Math., 3, North-Holland, Amsterdam, 1968.

[12] Grothendieck, A. *Groupes de Monodromie en Géométrie Algébrique.* Séminaire de Géométrie Algébrique du Bois-Marie 1967-69, SGA 7 I.

[13] Grothendieck, A. *Cohomologie ℓ-adique et Fonctions L.* Séminaire de Géométrie Algébrique du Bois-Marie 1965-66 (SGA 5). Ed. Luc Illusie. Lecture Notes in Mathematics 589, Springer-Verlag, 1977.

[14] Grothendieck, Alexandre. *Récoltes et Semailles I, II. Réflexions et témoignage sur un passé de mathématicien.* Galllimard, 2022.

[15] A. Grothendieck. *Résidus et dualité—Prénotes pour un "Séminaire Hartshorne".* Edited by Robin Hartshorne. Documents Mathématiques (Paris), 21. Société Mathématique de France, Paris, 2024.

[16] Hartshorne, Robin. *Residues and duality.* Lecture notes of a seminar on the work of A. Grothendieck, given at Harvard 1963/64. With an appendix by P. Deligne. Lecture Notes in Mathematics, No. 20. Springer-Verlag, Berlin-New York, 1966.

[17] Illusie, Luc. *Complexe cotangent et déformations. I.* Lecture Notes in Mathematics, Vol. 239. Springer-Verlag, Berlin-New York, 1971.

[18] Langlands, R. P. *Modular forms and ℓ-adic representations.* Modular functions of one variable, II (Proc. Internat. Summer School, Univ. Antwerp, Antwerp, 1972), pp. 361–500, Lecture Notes in Math., Vol. 349, Springer, Berlin-New York, 1973.

[19] Laumon, Gérard. *Homologie étale.* Séminaire de géométrie analytique (École Norm. Sup., Paris, 1974–75), pp. 163–188, Astérisque, No. 36-37, Soc. Math. France, Paris, 1976.

[20] Raynaud, Michel. *Caractéristique d'Euler-Poincaré d'un faisceau et cohomologie des variétés abéliennes.* Séminaire Bourbaki, no. 9 (1966), Exposé no. 286, in Dix exposés sur la cohomologie des schémas, 12–30, Adv. Stud. Pure Math., 3, North-Holland, Amsterdam, 1968.

[21] Serre, Jean-Pierre. *Représentations linéaires des groupes finis.* Première édition, Hermann, Paris, 1967. Troisième édition corrigée, Hermann, Paris, 1978.

[22] Stallings, John. *Centerless groups—an algebraic formulation of Gottlieb's theorem.* Topology 4 (1965), 129–134.

[23] Verdier, Jean-Louis. *A duality theorem in the etale cohomology of schemes.* Proc. Conf. Local Fields (Driebergen, 1966), pp. 184–198, Springer, Berlin-New York, 1967.

[24] Verdier, Jean-Louis. *The Lefschetz fixed point formula in etale cohomology.* Proc. Conf. Local Fields (Driebergen, 1966), pp. 199–214, Springer, Berlin-New York, 1967.

[25] Verdier, Jean-Louis. *Classe d'homologie associée à un cycle.* Séminaire de géométrie analytique (École Norm. Sup., Paris, 1974–75), pp. 101–151, Astérisque, No. 36-37, Soc. Math. France, Paris, 1976.