PROGRAM

Organizers

Center for Theoretical Physics Polish Academy of Sciences
Laboratoire de mathématiques d'Orsay, Université Paris-Saclay
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Speakers

Andrei Agrachov (SISSA) *Dynamic planimetry*
Abstract: This is a light talk. We consider segments and triangles on the plane and allow them to be deformed according to certain simple rules. This leads to few nice models of subriemannian spaces.

Ian Anderson (Utah State University) *Recent development in* **DG** *Software*
Abstract: In this series of talks I will present some new functionalities of the DG software package which are related to the theme of the workshop. Topics include:

(1) Differential Operators – symmetries, higher order symmetries, invariant differential operators, Lax pairs.
(2) Conformal Tractors – connections, parallel tractors and conformal Einstein space, G_2 conformal holonomy, reducible conformal holonomy.
(3) Generalized Flag Varieties – infinitesimal group actions, invariant distributions, filtered symmetry groups, double fibrations, applications from PDE.
(4) Connections on Principle Bundle – a gentle introduction, Yang-Mills instantons, reduction of the self-dual Yang-Mills equations.
(5) Time permitting – an example of Cartan reduction using the AbstractFrame environment in DG.

Gil Bor (CIMAT) *Dancing pairs, rolling balls and the Cartan-Engel distribution*
Abstract: A pair of planar polygons is "dancing" if one is inscribed in the other and they satisfy a certain cross-ratio relation at each vertex of the circumscribing polygon, as in the left figure. The middle figure shows a dancing pair of hexagons.

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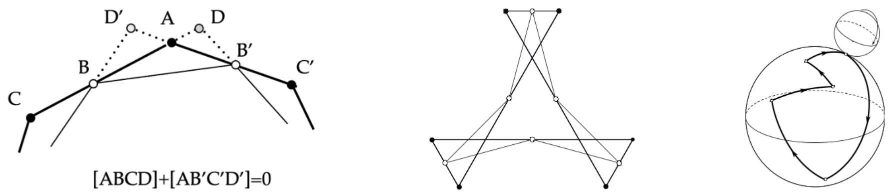

Dancing pairs correspond to trajectories of a non-holonomic mechanical system, consisting of a ball rolling, without slipping and twisting, along a polygon drawn on the surface of a sphere 3 times larger than the rolling ball. For example, the above dancing pair of hexagons corresponds to rolling twice around the perimeter of a right-angled equilateral triangle (an 'octant'). The correspondence stems from reformulating both systems as piecewise rigid curves of a certain remarkable rank 2 non-integrable distribution defined on a 5-dimensional quadric in 6-dimensional projective space, introduced by Cartan and Engel in 1893. The automorphism group of this distribution is the 14-dimensional Lie group G_2, and the above correspondence defines unexpected G_2-actions on the configuration spaces of both problems. This is joint work with Luis Hernández-Lamoneda, NY J Math 29, 2023, `https://arxiv.org/pdf/2304.07694.pdfarxiv.org/abs/2304.07694`.

Maciej Dunajski (DAMPT) *Quasi—Einstein metrics*
Abstract: We prove that the intrinsic Riemannian geometry of compact cross-sections of any Einstein extremal horizon must admit a Killing vector field. This establishes the rigidity theorem for the extremal Kerr black hole horizon.

This extremal horizon is a special case of a quasi-Einstein structure. We shall discuss other examples of quasi-Einstein structures on compact surfaces, including a global result relevant in projective metrizability.

Michael Eastwood (University of Adelaide) *Killing tensors on complex projective space*
Abstract: The Killing tensors on the round sphere are well understood, thanks to the methods of parabolic geometry: the round sphere is projectively flat so there is a Cartan connection et cetera, et cetera. In particular, the space of Killing tensors with some fixed rank is an irreducible representation of the appropriate special linear group acting by projective symmetries. What about the corresponding story for complex projective space with its Fubini-Study metric? Although this is seemingly not a parabolic geometry in any useful way, one can still muddle through. Some serious puzzles remain!

Jerzy Kijowski (CFT PAN) *Connection on a differential manifold: Newton versus Ehresman-Koszul-Nomizu. What is gravity?*
Abstract: The connection on a manifold, understood as a connection in the tangent bundle (or equivalently: in the principal "frame bundle"), is not irreducible, but is a pair of different objects: 1) a symmetric connection and 2) a (torsion) tensor. It is useful to interpret a symmetric connection as an autonomous object: a field of local inertial frames. An original theory of its curvature is presented. This is a modern version of Newton's fundamental physical idea. It has been shown that the very concept of a gravitational field can be identified with the field of inertial

frames: no metric structure is necessary here. This approach to gravity theory reproduces the General Relativity Theory, but also offers new tools to properly describe large-scale effects (dark matter, etc.).

Boris Kruglikov (UiT the Arctic University of Norway) *Invariant divisors and equivariant line bundles*
Abstract: Analytic hypersurface of a complex manifold M is described by a divisor. A divisor gives rise to a line bundle over M, and the corresponding invariant hypersurface is described by the zero locus of a nontrivial section of this bundle. We consider a Lie algebra g represented by vector fields on M and look for g-invariant divisors, a global counterpart to the classical notion of scalar relative invariants. We discuss how the invariant divisors relate to g-equivariant line bundles, and show several aspects of the theory of invariant divisors with computational examples. In this way we relate various versions of the Picard group with a double complex, interpolating Čech and Chevalley-Eilenberg. A Lie group version of this theory is also available. We finish with applications to the global theory of relative differential invariants and classification of invariant differential equations.
This is joint work with Eivind Schneider.

Wojciech Kryński (IMPAN) *Lewy's curves in 3-dimensional para-CR geometry*
Abstract: We study a class of path geometries canonically associated with any given para-CR manifold in dimension 3. The path geometries are defined by curves that are natural counterparts of the Lewy's curves studied in CR-geometry by J.Faran (Trans. AMS, 1981). We characterize the geometries in terms of the invariants of systems of ODEs. Relations to the dancing construction are presented as well.

Ben McKay (University College Cork) *Automorphism groups of Cartan geometries*
Abstract: The automorphism group of a Cartan geometry is a Lie group, as is well known. I will talk about the errors in the known proofs of that fact and explain how to fix them. I will explain why the total space of the Cartan geometry quotients by the automorphism group to a smooth orbit space, as a principal bundle.

Lionel Mason (University of Oxford) *Global split signature self-dual Einstein metrics and their hidden symmetries*
Abstract: This lecture gives a geometric understanding of global self-dual Einstein metrics in split signature that are asymptotically hyperbolic. A particular class of such solutions are shown to be in one-to-one correspondence to deformations of the real slice $\mathbb{RP}^3$ of of complex projective 3-space $\mathbb{CP}^3$ on which a certain contact structure is real. The space-time is reconstructed as a moduli space of holomorphic discs with boundary on the real slice. The space of such Einstein metrics is a homogeneous space for holomorphic Poisson diffeomorphisms defined near the real slice, and can be explicitly represented in terms of generating functions. The Poisson diffeomorphism play a role as a hidden symmetry algebra as far as space-time is concerned and arise in the holographic study of gravitational amplitudes and correlation functions. This can be understood via a variational problem for the

holmorphic discs.

Nefton Pali (Universite Paris-Sud) *On maximal totally real embeddings*
Abstract: In a joint article with Bruno Salvy, we consider almost complex structures with totally real zero section of the tangent bundle. We assume that the almost complex structure tensor is real analytic on the fibers of the tangent bundle.

This hypothesis is very natural in view of a well-known result of Bruhart and Whitney on the existence of complex structures over Grauert Tubes. We provide explicit integrability equations for such almost complex structures in terms of the Taylor expansion on the fiber.

For any torsion-free connection acting on the real analytic sections of the tangent bundle of a real analytic manifold, we provide a very simple and very explicit expression of the coefficients of the Taylor expansion on the fiber of the associated canonical complex structure.

Our expression removes the deep mystery lasts more than 64 years on the explicit global form of complex structures on Grauert Tubes since the work of Bruhat-Whitney.

An explicit global expression for the above coefficients is important for applications to analytic micro local analysis over manifolds, as it allows an explicit global construction of the complex extension of a given global Fourier integral operator defined on a real analytic manifold.

Jean Petitot (Centre d'Analyse et de Mathématiques Sociales, École des Hautes Études) *The primary visual cortex as a Cartan engine*
Abstract: Cortical visual neurons detect very local geometric cues as retinal positions, local contrasts, local orientations of boundaries, etc.). One of the main theoretical problem of low level vision is to understand how these local cues can be integrated so as to generate the global geometry of the images perceived, with all the well-known phenomena studied since Gestalt theory. It is an empirical evidence that the visual brain is able to perform a lot of routines belonging to differential geometry. But how such routines can be neurally implemented ? Neurons are 'point-like' processors and it seems impossible to do differential geometry with them. Since the 1990s, methods of "in vivo optical imaging based on activity-dependent intrinsic signals" have made possible to visualize the extremely special connectivity of the primary visual areas, their "functional architectures." What we called 'Neurogeometry' is based on the discovery that these functional architectures implement structures such as the contact structure and the sub-Riemannian geometry of jet spaces of plane curves. For reasons of principle, it is the geometrical reformulation of differential calculus from Pfaff to Lie, Darboux, Frobenius, Cartan and Goursat which turns out to be suitable for neurogeometry.

Witold Respondek (Łódź University of Technology, Poland and INSA Rouen, France) *Geometry of linearizable mechanical control systems*
Abstract: For mechanical control systems we discuss the problem of linearization that preserves the mechanical structure of the system. We give necessary and sufficient conditions for both the mechanical state-space-linearization and mechanical feedback-linearization using geometric tools, like covariant derivatives, symmetric

brackets, and the Riemann tensor, that have an immediate mechanical interpretation. In contrast with linearization of general nonlinear systems, conditions for their mechanical counterpart can be given for both, controllable and noncontrollable, cases. We illustrate our results by examples of linearizable mechanical systems. The talk is based on joint research with Marcin Nowicki (Poznan University of Technology, Poland).

Adam Sawicki (CFT PAN) *Quantum correlations and geometry*
Abstract: During this talk I will discuss how various geometric concepts can be used to quantify quantum correlations in many body systems. In particular I will explain the role played by symplectic structures and momentum maps. In the last part of my talk I will also speak about an ongoing recently started project (in collaboration with F. Costanza, P. Nurowski and B. Sikorski) which focuses on quantum correlations in fermionic systems with non-fixed numbers of particles.

Stefan Suhr (Ruhr Universität–Bochum) *Pseudo-Riemannian Zoll manifolds*
Abstract: In this easy going talk I will report on a beautiful but rich geometry subject; Zoll manifolds in pseudo-Riemannian geometry. Classically Zoll manifolds are Riemannian manifolds all of whose geodesics are closed and of the same minimal period. It is an extended theory (see the book by A. Besse) which resurfaces every couple of years with new results. On the other side the same question for pseudo-Riemannian metrics is rather underdeveloped. I will give an overview of what is known for surfaces and if time permits say a few words about 3-dimensional Zollfrei manifolds following a conjecture of Guillemin.

Dennis The (Arctic University of Norway) *On 4D split-conformal structures with G2-symmetric twistor distribution*
Abstract: In their 2014 article, An & Nurowski considered two surfaces rolling on each other without twisting or slipping, and defined a twistor distribution (on the space of all real totally null self-dual 2-planes) for the associated 4D split-signature conformal structure. If this split-conformal structure is not anti-self dual, then the twistor distribution is a (2,3,5)-distribution, and An-Nurowski identified interesting rolling examples where it achieves maximal, i.e. G2, symmetry. Relaxing the rolling assumption, a similar construction can be made for any 4D split-conformal structure, and my talk will discuss a broader classification of examples where such exceptional symmetry for the twistor distribution is achieved. (Joint work with Paweł Nurowski & Katja Sagerschnig.)

The organizers, **Joël** and/or **Paweł**, may also talk if there is any interest in talks about e.g. accidental CR (or para-CR) structures and their relations to the variopus real forms of the exceptional simple Lie algebras, or about musical tunings and plans to build a totally new acoustic piano (not neccessarily *decaphonic*).