# Control, index, traces and determinants

*the journey of a probabilist*

Michael Atiyah ( Edinburgh )

**Branched coverings and $K$-theory**

Much work has been done on quotients of manifolds by finite groups, in particular in connection with index theory. The case when the group is cyclic and has fixed point set of codimension 2 is especially simple, since the quotient is itself a manifold. For this reason there are extra results that can be obtained and the proofs are remarkably easy when approached the right way.

Nicolas Bergeron ( Paris 6 )

**On the Hodge conjecture for compact arithmetic quotients of complex balls**

Let $S$ be a closed Shimura variety uniformized by the complex $n$-ball. The Hodge conjecture predicts that every Hodge class in $H^{2k}(S,Q)$, $k=0,\ldots,n$, is algebraic. In this talk I will describe some of the ideas of the proof that the Hodge conjecture holds for all degree $k$ away from the neighborhood $]n/3,2n/3[$ of the middle degree. This is joint work with John Millson and Colette Moeglin.

Simon Donaldson ( Imperial College )

**Volume estimates, Chow invariants and moduli of Kähler–Einstein metrics**

We outline a proof of the result that the existence of a Kähler–Einstein metric is a Zariski-open condition. This has recently been established by Odaka, as a consequence of the result of Chen, Sun and the speaker, connecting this existence with "$K$-stability". The proof we discuss here is different and has the advantage that it extends to constant scalar curvature metrics under certain additional assumptions. The central idea is to estimate Chow invariants using the asymptotic analysis of the Bergman kernel and estimates on the volumes of neighbourhoods of points where the Riemannian curvature is large.

Julien Dubédat ( Columbia )

**Dimers and families of Cauchy-Riemann operators**

Dimers are a classical model in combinatorics and statistical mechanics. In the plane, their large scale asymptotics are governed by the (compactified) free field. We discuss several features of the model from the point of view of variational analysis of families of Cauchy-Riemann operators (and their finite difference approximations).

Yasha Eliashberg ( Stanford )

**Topology of polynomially and rationally convex domains.**

I will discuss in the talk necessary and sufficient conditions for a domain in $\mathbb{C}^n$, $n > 3$, to be isotopic to polynomially, or rationally convex domain. This result, which is a joint work with Kai Cieliebak is an application of recent advances in symplectic flexibility.

Gerd Faltings ( Bonn )

**Norm of Weierstrass sections**

Weierstrass divisors are an important tool in diophantine geometry, as they give canonical sections of powers of the dualising bundle (on curves). We give estimates for the archimedean norm and (in case of bad reduction) also of the p-adic norm.

Ezra Getzler ( Northwestern )

**Higher analytic stacks, twisted complexes of holomorphic vector bundles, and the definition of the determinant**

Sebastian Goette ( Freiburg )

**Morse functions and families torsion**

We sketch the construction of torsion forms for families of compact manifolds using generalised fibrewise Morse functions. These torsion forms have similar properties as the Bismut–Lott torsion forms, and they can be used to give another proof of the Bismut–Lott index theorem for flat vector bundles. On the other hand, they give rise to an extension of the Igusa–Klein torsion classes.

Maxim Kontsevich ( I.H.É.S. )

**On Kähler random walk and Lyapunov exponents**

John Lott ( Berkeley )

**Transverse index problem for Riemannian foliations**

It is still an open problem to give a local formula for the index of a transverse Dirac-type operator on a manifold with a Riemannian foliation. I will give an answer in the first nontrivial case, namely when the Molino Lie algebra of the foliation is abelian. This is joint work with Alexander Gorokhovsky.

Raffe Mazzeo ( Stanford )

**Asymptotics of special metrics: methods and applications**

I will describe various results centering around the theme of how to obtain detailed asymptotics of special metrics on noncompact or singular spaces, and specific uses of this information. The main examples in this talk include the Weil–Petersson metric on the Riemann moduli space with applications to spectral geometry on this space, the role of regularity theory near singular divisors in an existence theorem for Kähler-Einstein metrics, and the asymptotic geometry of Hitchin's Higgs bundle moduli spaces.

Richard Melrose ( M.I.T. )

**Fusive loop-spin structures**

In the 1980s Witten exploited the transgression relationship between a manifold and its loop space to derive the formal index for the Dirac-Ramond operator, on the loop space, so introducing his genus, with integality properties arising from a String structure on the manifold. In joint work with Chris Kottke, as a first step towards actually analyzing the Dirac-Ramond operator, loop-spin structures with good properties are shown to be in 1-1 correspondence with String structures; the latter having been classified by Redden. This extends and refines recent results of Waldorf, based in turn on the introduction by Stolz and Teichner of the notion of 'fusion' which elevates transgression to an isomorphism.

Werner Müller ( Bonn )

**Analytic torsion of locally symmetric spaces and its application to arithmetic groups**

Analytic torsion is a spectral invariant of a compact Riemannian manifold and a flat vector bundle over it. It is defined as a weighted product of regularized determinants of the Laplacians on forms twisted by the flat bundle. In this talk I will consider the analytic torsion of locally symmetric spaces of finite volume. The main purpose is to determine its asymptotic behavior with respect to either special sequences of flat vector bundles or sequences of finite coverings. This has applications to the study of the growth of torsion in the cohomology of arithmetic groups. I will review some of the recent results and discuss further problems.

Damian Rössler ( Toulouse )

**On the Bismut–Koehler analytic torsion form of the Poincaré bundle of abelian schemes.**

The Bismut–Koehler analytic torsion form of the Poincaré bundle on an abelian scheme has several remarkable properties. For instance, it is compatible with isogenies (this is a "distributivity property") and when restricted to a torsion section, it lies in the image of the regulator map of the base. If the abelian scheme has an arithmetic model, it even lies in the image of the regulator map of the arithmetic model of the base. In the case of elliptic schemes, it reduces to the ordinary Ray–Singer analytic torsion and it is given by a Siegel function. We shall give an overview of these properties and explain how the arithmetic Grothendieck–Riemann–Roch theorem of Bismut–Gillet–Soulé can be used to establish them.

Scott Sheffield ( M.I.T. )

**Random surfaces: real and imaginary**

Andrei Teleman ( Université de Provence )

**Determinant line bundles in non-Kählerian geometry and instanton moduli spaces over class VII surfaces**

We use recent results of Jean-Michel Bismut on the curvature of the determinant line bundle in non-Kählerian geometry to study a non-Kählerian version of the Fourier-Mukai transform. We apply these ideas and gauge theoretical techniques to describe certain instanton moduli spaces on minimal class VII surfaces. We show how our instanton moduli spaces can be used to prove existence of curves on these surfaces.

Gang Tian ( Princeton & Peking University )

**Ricci flow on Fano manifolds**

Cédric Villani ( Lyon I )

**Hypocoercivity: results and problems**

Wendelin Werner ( Université Paris-Sud )

**Loops, fields and carpets**

Ken-Ichi Yoshikawa ( Kyoto )

**Equivariant analytic torsion for K3 surfaces with involution**

There are 75 distinct families of $K3$ surfaces with anti-symplectic holomorphic involution. We recall the construction of the invariant of those $K3$ surfaces using equivariant analytic torsion and its automorphic property as a function on the moduli space. Then we discuss the structure of the corresponding automorphic form on the moduli space. Our main result is that, except for possible 7 families, the automorphic form is the product of a Borcherds product and an explicitly given section of the Hodge bundle on the moduli space of curves. If time permits, some explicit formulas shall also be given.