On Mathematical Definition of Chords between Networks

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Abstract— **This paper deals with a tentative to define mathematically the "chords" used in the tensorial analysis of networks (TAN), usually employed by electricians to describe some magnetic interaction in transformers for example, and more generally in the TAN to describe any interaction between branches or meshes. While the formalism is well accepted by the users of the TAN method, it appears that it seems there is no mathematical definition for these objects. In order to insert the TAN in a general mathematical context including algebraic topology and graph theory, we try to propose a rigorous definition of these chords. Firstly we briefly recall the TAN method then the chords concept. Secondly we place the concept in the category context. In this theory, the chords appear to be the functors linking between various connex but separate networks. In conclusion we speak of futures works always in the idea to enforce the links between the TAN method and the algebraic topology.**

Keywords-component; tensorial analysis of networks, topology, categories, functors, chords.

INTRODUCTION

The TAN method, elaborated by Sir Gabriel Kron in 1939, (perhaps the second father of topology applied to electromagnetism after Sir Gustav Kirchhoff - †1887) has demonstrated its capability to model complex electromagnetism situation like electrical machine [1] and recently, electromagnetic compatibility [2]. Banesh Hoffmann has already discussed of the Kron's approach in his well-known "Kron's Non-Riemannian Electrodynamics" [3]. In his symbolic representation for machines and in general for transformers, Kron uses symbolic lines to represent mutual inductances between coils [4]. Our purpose is to try to place this symbolic and pragmatic approach in a more fundamental definition reported in the discrete algebraic topology theory used e.g. in Rapetti et al [5].

TAN PRINCIPLES

A. *Structures*

The TAN is based on the graph theory as graphic structure and the tensorial algebra as algebraic structure. Confine electromagnetic energy exchanges can be represented by flux on graphs on a discrete topology usually noted C^n. Connex graphs involve nodes, branch (or edges) and meshes (or cycles) referring to C^0 and C^1 spaces in C^n. Electric interactions are projected on these connex graphs where each branch is a manifold with a straight link with the known physical properties of the electric field in space. Classical Kirchhoff's laws can be retrieve through this discrete topology formalism.

B. *Mutual inductances*

Another graph can be constructed for the magnetic interactions, involving magnetic flux and magnetomotive forces. This description involves the C^2.

C. *Hamiltonian cross talk*

To couple both electric and magnetic representations, we can easily use the flux derivation. But formally, this operation needs to speak of a Hodge operator [5] as both graph doesn't belongs to the same C^n spaces. To create the link, we give themselves a metric – the magnetic permeability - which will define as a consequence the electric permittivity and conductivity as metrics too. This mechanism and the Tonti's diagrams of each situation describe the mecanisms involved.

D. *Lagrangian description*

In order to reduce the hamiltonian two coupled equations in a single one – a lagrangian description – we need to invent a special relation between the meshes: the mutual inductance or in other word, a "chord" added to link two connected components of the network. This chord can be seen as a functor if each network is considered as a category of "Top" kind, the chord making the link between a mesh of the first network to a mesh in the second network. This functor preserves the connection group between the branch and the mesh space.

CONCLUSION

The extension of the functor to any interaction between branches or meshes from two different networks is immediate, including the ones delaied coming from non confine interactions like the radiated far field. In this case, the discrete topology appears as a tangent plane to the continuous manifold of the Maxwell's fields where a moment space is the vector space attached to the affine one of C^n[6].

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