

# Equivalent partial differential equations and applications

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with a special thanks to Pierre Lallemand

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analysis of multiple relaxation times lattice Boltzmann schemes  
with the Taylor expansion method and the ABCD approach:  
generalization of the Chapman Enskog methodology



inverse problem for Navier Stokes  $\Phi(W) = ?$ ,  $S = ?$

isothermal Navier Stokes

D3Q27 has a discrepancy for isothermal Navier Stokes



D3Q27-2 available for isothermal Navier Stokes



thermal Navier Stokes

we must impose  $\gamma \equiv \frac{c_p}{c_v} = 2$  (2d),  $\gamma = \frac{5}{3}$  (3d)

and a Prandtl number satisfying  $Pr = 1$



stability not been studied



exact solution of cosine advection operator with Fourier series

be careful with the initialization order !



coherent convergence order for finite time evolution





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thank you for your attention!

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