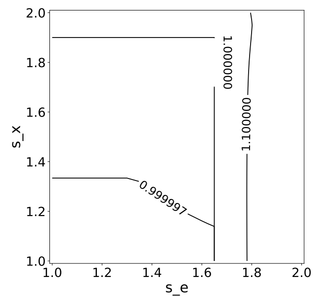
MRT lattice Boltzmann schemes with projection

François Dubois[1*] and Paulo Cesar Philippi[2]
[1] LMSSC, Conservatoire National des Arts et Métiers, Paris and LMO Orsay, France.
francois.dubois@universite-paris-saclay.fr
[2] Pontifical Catholic University of Paranà, Curitiba, Brazil.

Key words: *Stability; D2Q9. Submitted the 20 March 2024 to the DSFD-2024 conference.*
Method 01-Lattice Boltzmann Method
Area of application A-Method Development

We have observed in [1] that three families of moments emerge from the asymptotic analysis of lattice Boltzmann schemes: the conserved moments W that define the unknowns of the equivalent partial differential equations, the nonconserved Eulerian moments Y_e for setting the first order terms and the nonconserved viscous moments Y_v for adjusting second-order dissipation. The equilibrium values for Eulerian and viscous moments are denoted by $Y_e^{\rm eq}$ and $Y_v^{\rm eq}$ respectively. The relaxation of Eulerian moments satisfies a relation of the type $Y_e^* = Y_v + S_e\,(Y_e^{\rm eq} - Y_e)$ with a diagonal relaxation matrix S_e parametrized by the two coefficients s_e and s_v for the D2Q9 scheme. Instead of the classical relaxation $Y_v^* = Y_v + S_v\,(Y_v^{\rm eq} - Y_v)$ for the viscous moments [2], we propose here an algorithm inspired by kinetic regularization involving Hermite polynomials (see *e.g.* [3, 4, 5] and many others!).
We first introduce two matrices $K(W)$ and $L(W)$ that depend only on the conserved variables and satisfying the following constraints for the equilibria: $Y_v^{\rm eq} \equiv K(W)\,W + L(W)\,Y_e^{\rm eq}$. Then the viscous moments are projected onto conserved moments and Eulerian moments: $P\,Y_v = K(W)\,W + L(W)\,Y_e$. In the MRT lattice Boltzmann schemes with projection, the relaxation step $Y_v \longrightarrow Y_e^*$ is replaced by the projection followed by the relaxation of the Eulerian moments: $Y_v \longrightarrow (PY_v)^*$ with $(PY_v)^* = K(W)\,W + L(W)\,Y_e^*$.
We have implemented this algorithm for the D2Q9 scheme. First, the asymptotic analysis shows that up to second order accuracy, the equivalent partial differential equations are identical to those obtained with the the initial lattice Boltzmann scheme. Secondly, our preliminary results concern a linearized version of the D2Q9 scheme aroud a constant state with velocity u_0 and sound velocity $c_0 = \frac{1}{\sqrt{3}}$. They show a significant increase in the stability zone for the two relaxation parameters (see Figure 1).



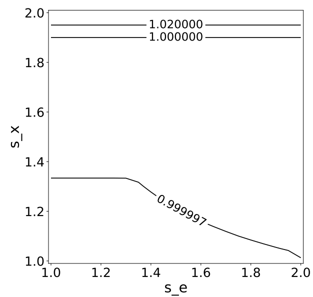


Figure 1: Comparison of linear stability zones for advection speed $u_0 = 0.35\,c_0$. Traditional D2Q9 scheme [2] on the left and D2Q9 MRT with projection on the right.

References

[1] F. Dubois, P. Lallemand, *Communications in Computational Physics*, **34**, 613-671, 2023.
[2] P. Lallemand, L.-S. Luo, *Physical Review E*, **61**, 6546-6562, 2000.
[3] J. Latt, B. Chopard, *Mathematics and Computers in Simulation*, **72**, 165-168, 2006.
[4] O. Malaspinas, arXiv:1505.06900v1, 2015.
[5] K. K. Mattila, P. C. Philippi, L. A. Hegele Jr, *Physics of Fluids*, **29**, 46103 (13 pages), 2017.