Lattice Boltzmann schemes with relative velocity

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Some theoritical aspects of the Lattice Boltzmann Method remain debatable : stability for low viscosities and lack of Galilean invariance are two major issues of this kind of scheme.
Geier proposed in 2006 a cascaded Boltzmann scheme for which all the equilibria are constant in term of the fluid velocity [1]. He used the triangular structure of a matrix to relax to these equilibria. It seems to bring a gain on those issues.
His scheme induces us to define a new class of lattice Boltzmann method depending on a velocity parameter which is a function of space and time. Note that this method is an adaptation of the Geier scheme and the d'Humières scheme [2]. The idea is to relax in the moments space that moves at this velocity. We expect to obtain similar results to Geier's ones by fixing the parameter as the fluid velocity. This method does not add any difficulty to the implementation and provides a triangular structure during the change of frames step the fixed frame to the moving frame or the opposite.
Moreover, we are interested in the choice of the moments as we want to get a group representation for the change of frames. This requirement implies to have a well-choosen family of polynoms for those moments.
We also derive equivalent equations up to the third order using the Taylor expansion method. These equations do not depend on the velocity parameter up to the second order so that our scheme approximate the same physics as classical DdQq schemes. We especially develop completely the equations of the non linear D2Q9 scheme up to the second order to exhibit non galilean invariant terms.
We also propose some numerical tests to compare the properties of velocity dependent DdQq schemes with classical DdQq schemes.

References

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