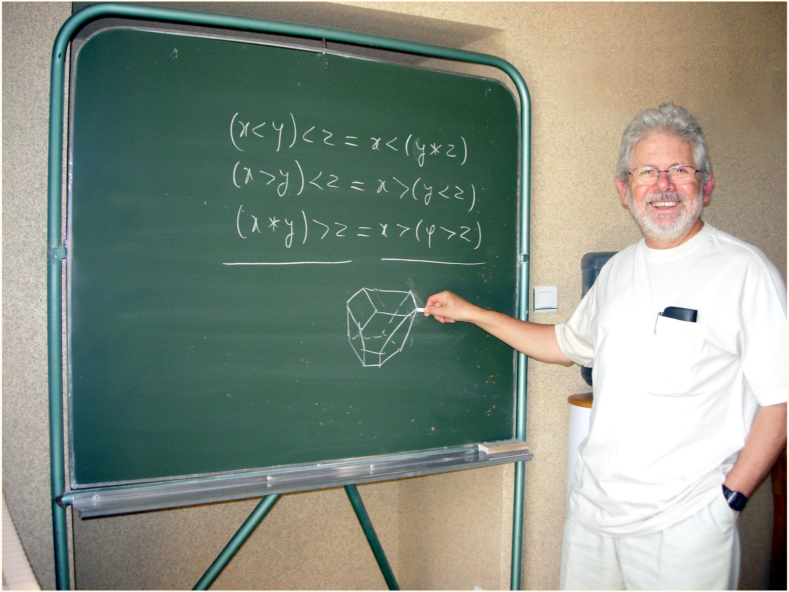
Recovering the full Navier Stokes equations with lattice Boltzmann schemes

François Dubois[*,†], Benjamin Graille[†] and Pierre Lallemand[‡,§]

NUMKIN 2016: International Workshop on Numerical Methods for Kinetic Equations
Strasbourg, 20 october 2016

* Conservatoire National des Arts et Métiers, Paris, France
† Université Paris-Sud, France
‡ Centre National de la Recherche Scientifique, Paris, France
§ Beijing Computational Science Research Center, China

Jean-Louis Loday (1946 - 2012)



From Boltzmann equation to lattice Boltzmann schemes

Boltzmann equation $\partial_t f + v\bullet\nabla f = Q(f)$

Linearization $Q(f) \simeq Q(f^{\mathrm{eq}}) + \mathrm{d}Q(f^{\mathrm{eq}})\bullet(f - f^{\mathrm{eq}})$

Boltzmann BGK $\partial_t f + v\bullet\nabla f = \mathrm{d}Q(f^{\mathrm{eq}})\bullet(f - f^{\mathrm{eq}})$

Discrete velocities

$$\partial_t(Mf) + M\bullet v\bullet\nabla f = \big(M\,\mathrm{d}Q(f^{\mathrm{eq}})\,M^{-1}\big)\bullet\big(M\,(f - f^{\mathrm{eq}})\big)$$

Moments $m = M\,f\,,\quad m^{\mathrm{eq}} = M\,f^{\mathrm{eq}}$

Multiple Relaxation Times hypothesis :

the matrix $M\,\mathrm{d}Q(f^{\mathrm{eq}})\,M^{-1}$ is diagonal and real

Time discretization: alternate directions, or “collide-stream”

Zero eigenvalues $\frac{\mathrm{d}m_k}{\mathrm{d}t} = 0$, $0 \leq k < N$: conserved moments W

Relaxation of the nonconserved moments m_k, $k \geq N$

$\frac{\mathrm{d}m_k}{\mathrm{d}t} = -\frac{1}{\tau_k}\left(m_k - m_k^{\mathrm{eq}}\right)$, $m_k^* = m_k - \frac{\Delta t}{\tau_k}\left(m_k - m_k^{\mathrm{eq}}(W)\right)$, $s_k = \frac{\Delta t}{\tau_k}$

Particle densities $f_j^*(x, t) = \left(M^{-1}\,m^*\right)_j(x, t)$

$x \in \mathcal{L}\mathrm{attice}$ and $x + v_j\,\Delta t \in \mathcal{L}\mathrm{attice}$, $t = n\,\Delta t$

Solve the advection equation $\partial_t f_j + v_j\bullet\nabla f_j = 0$ with CFL=1!

$$f_j(x + v_j\,\Delta t,\ t + \Delta t) = f_j^*(x,\ t)$$

Lattice Boltzmann scheme DdQq

Outline

- Fluid flow with a Multi Relaxation Times
  D1Q3 lattice Boltzmann scheme
  Advection-diffusion with a Multi Relaxation Times
  D1Q3 lattice Boltzmann scheme
- Double distribution for Navier Stokes
  Navier Stokes with mass, momentum and total energy
  Double distribution with the Bhatnagar Gross Krook
  lattice Boltzmann framework
- Navier Stokes with mass, momentum and volumic entropy
  Double distribution for a Multi Relaxation Times
  D1Q3Q3 lattice Boltzmann scheme
  Linearized study with a focus on stability
- Definition of a nonlinear scheme
  First numerical experiments

Thanks for your attention !