On anti bounce back boundary condition *

François Dubois[a], Pierre Lallemand[b] and Mohamed-Mahdi Tekitek[c]

[a] *CNAM-Paris, LMSSC, and Dpt. Mathematics, Univ. Paris-Sud, Orsay, France.*

[b] *Beijing Computational Science Research Center, Haidian District, Beijing 100094, China.*

[c] *Dpt. Mathematics, Faculty of Sciences of Tunis, University Tunis El Manar, Tunis, Tunisia.*

In [1, 2], we have proposed to extend the Taylor expansion method [4] for the analysis of the bounce back boundary condition in the particular case of the d2q9 lattice Boltzmann scheme [3] for a bottom boundary. In this contribution, we apply the same methodology to analyze so so-called "anti bounce back" boundary condition [5], a standard procedure to impose a pressure boundary condition.

We consider the linear d2q9 scheme with a particle distribution f and associated moments $m \equiv M\, f$ classically denoted as ρ, J_x, J_y, e, XX, XY, q_x, q_y, ε. The space-time lattice with space step Δx and time step Δt defines the mesh velocity $\lambda \equiv \frac{\Delta x}{\Delta t}$. The three first moments are conserved. The equilibrium values of the nonconserved moments satisfy $e^{\rm eq} = \alpha\, \lambda^2\, \rho$, $XX^{\rm eq} = 0$, $XY^{\rm eq} = 0$, $q_x^{\rm eq} = -\lambda^2\, J_x$, $q_y^{\rm eq} = -\lambda^2\, J_y$ and $\varepsilon^{\rm eq} = \beta\, \lambda^4\, \rho$.

At a right boundary at abscissa L where the density ρ_L is given, and in consequence the pressure due to the relation $p \equiv \frac{\alpha+4}{6}\, \lambda^2\, \rho$, the anti bounce back boundary condition expresses that the incoming particles f_3, f_6 and f_7 are simple affine functions of the corresponding outgoing particules $f_1^*(x)$, f_8^* and f_5^* after relaxation:

$f_3(x, t+\Delta t) = -f_1^*(x) + \frac{1}{18}\, (\alpha - 2\, \beta - 4)\, \rho_L$, $f_6(x, t+\Delta t) = -f_8^*(x) + \frac{1}{18}\, (2\, \alpha + \beta + 4)\, \rho_L$, $f_7(x, t+\Delta t) = -f_5^*(x) + \frac{1}{18}\, (2\, \alpha + \beta + 4)\, \rho_L$.

As in [1, 2], we represent the time iteration of the lattice Boltzmann scheme with an algebraic expression taking into account both internal and boundary schemes and we conduct the analysis with the methodology developed previously. We obtain a description of the anti bounce back boundary condition at various orders. Technically, we have to solve a ill-posed linear system with a kernel of dimension 2. Two compatibility conditions are associated with this ill-posed linear system and the solution is defined up to a space of dimension 2. As a consequence, the conserved momentum $(J_x,\, J_y)$ remains undefined at order zero. At first order, one of the compatibility conditions can be expressed as $\partial_x J_y + \partial_y J_x = {\rm O}(\Delta t)$. We will emphasize during the conference how this last differential condition has to be interpreted.

References

[1] F. Dubois, P. Lallemand, M. Tekitek. ESAIM ProcS, vol. 52, p. 25-46, 2015.

[2] F. Dubois, P. Lallemand, M. Tekitek. Proceedings if the ICMMES-2016 conference, submitted to *Computers and Fluids*, 2017.

[3] P. Lallemand, L-S. Luo. *Physical Review E*, vol. **61**, p. 6546-6562, 2000.

[4] F. Dubois. *Computers and mathematics with applications*, vol. **55**, p. 1441-1449, 2008.

[5] D. d'Humières, I. Ginzburg. *Physical Review E*, vol. **68**, issue 6, p. 066614, 2003.

* Submitted the 13 April 2017 to the 14th ICMMES Conference, Nantes, 17 - 21 July 2017.