Curious convergence properties of lattice Boltzmann schemes for diffusion with acoustic scaling *

Bruce M. Boghosian[a], François Dubois[bc], Benjamin Graille[b], Pierre Lallemand[d] and Mohamed-Mahdi Tekitek[e]

[a] *Dpt. of Mathematics, Tufts University, Bromfield-Pearson Hall, Medford, MA 02155, U.S.A.*

[b] *Dpt. of Mathematics, University Paris-Sud, Bât. 425, F-91405 Orsay, France.*

[c] *Conservatoire National des Arts et Métiers, LMSSC laboratory, F-75003 Paris, France.*

[d] *Beijing Computational Science Research Center, Haidian District, Beijing 100094, China.*

[e] *Dpt. Mathematics, Faculty of Sciences of Tunis, University Tunis El Manar, Tunis, Tunisia.*

In [1], we have studied the asymptotic expansion of various lattice Boltzmann schemes with muti-relaxation times for different applications. We used the so-called acoustic scaling: the ratio $\lambda \equiv \Delta x / \Delta t$ is remained fixed. To present the problem in this abstract, we explain the situation for a very simple d1q3 scheme.

The moments ρ, J and e are constructed from the particle distribution f_+, f_0 and f_-: $\rho \equiv f_+ + f_0 + f_-$, $J \equiv f_+ - f_-$ and $e \equiv f_+ - 2\, f_0 + f_-$. The momentum ρ is conserved. The two other equilibria follow simple linear relations: $J^{\rm eq} = 0$ and $e^{\rm eq} = \alpha\, \rho$. The relaxation of these two nonconserved moments introduce two parameters s and s': $J^* = (1-s)\, J + s\, J^{\rm eq}$, $e^* = (1-s')\, e + s'\, e^{\rm eq}$. When s and s' are fixed, the density ρ follows approximatively a diffusion equation as the space step Δx tends to zero:

(1) $$\partial_t \rho \,-\, \mu\, \partial_x^2 \rho \,=\, {\rm O}(\Delta x^2)\,.$$

The diffusion coefficients μ is given by the relation

(2) $$\mu \,=\, \frac{2+\alpha}{3} \Big(\frac{1}{s} - \frac{1}{2} \Big)\, \lambda\, \Delta x\,.$$

Imagine now that we wish to approximate the diffusion equation (1) with the d1q3 lattice Boltzmann scheme described previously. We suppose that the diffusion coefficients μ is fixed and that the mesh size Δx tends to zero. Then from the relation (2), the relaxation parameter s cannot be no longer fixed and tends to zero according to the asymptotics

(3) $$s \,=\, \frac{2+\alpha}{3\,\mu}\, \lambda\, \Delta x \,+\, {\rm O}(\Delta x^2)\,.$$

The hypothesis done for deriving the diffusion model (1) (2) is now in defect. The relaxation coefficient s follows an asymptotics of the type

(4) $$s \,=\, s_0 + s_1\, \Delta x + s_2\, \Delta x^2 + \cdots$$

as suggested in [2]. Moreover, we have $s_0 = 0$ in the case described in (3).

In the conference on Discrete Simulation of Fluid Dynamics, we will present a complete analytical and numerical study for determining the asymptotic partial differential equations of this basic d1q3 lattice Boltzmann scheme. We will also show that a structure appears for the associated asymptotic equations when an hypothesis of the type (4) is satisfied for the relaxation parameters.

References

[1] F. Dubois, P. Lallemand. *J. Stat. Mech.: Theory and Experiment*, P06006, 2009.

[2] B. Boghosian, W. Taylor, *Phys. Rev. E*, vol. 52, p. 510-554, 1995.

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