FROM ISOTROPIC PARTIAL DIFFERENTIAL EQUATIONS TO LATTICE BOLTZMANN SCHEMES

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The lattice Boltzmann schemes (see [5]) are governed by a certain number of parameters that must be carefully chosen according to the applications. The objective of our work is to understand the influence of the choice of these parameters on the properties of isotropy of the solution until high orders. These isotropy properties are essential both for physical applications and for improving stability.

First, we present general results on isotropic partial differential equations. We explain how the isotropy property (invariance by rotation of the spatial frame) implies some characterization of the tensors that define the equations. Then, we introduce a methodology to define an isotropic lattice Boltzmann scheme thanks to the isotropy of partial differential equations. With the Taylor expansion method [4], it is possible to determine the equivalent partial differential equations of a lattice Boltzmann scheme. Then we adapt the methodology presented in [2] to make isotropic the partial equivalent equations of these schemes up to the order 4.

We apply this method among the most popular schemes in dimension 3: D3Q19 and D3Q27 (results on dimension 2 are given in [3]). More precisely, there exists a set of parameters that gives D3Q19 scheme isotropic up the third order without constraint on the physical parameters (viscosities and sound velocity remain free). However, the D3Q19 is isotropic at the fourth order only if the three physical data (shear viscosity, bulk viscosity and sound velocity) are linked. The D3Q27 scheme is a generalization of the D3Q19: there exists a set of parameters that give this scheme isotropic up to fourth order.

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