Towards perfectly matching layers for LBE *

Mahdi Tekitek[1], M'hamed Bouzidi[2], François Dubois[1], and Pierre Lallemand[2]

[1] *Laboratoire de Mathématiques, Université Paris Sud, Bat. 425, 91405 Orsay Cedex, France.*
[2] *Laboratoire ASCI, Université Paris Sud, Bâtiment 506, 91405 Orsay Cedex, France.*

The Bérenger method [1] consists in introducing a buffer domain (Perfectly Matching Layer, or PML) which surrounds the domain of interest. In this buffer domain the equations are modified with the aim of absorbing all waves incoming from the domain of interest with no reflected wave.

- **Construction of the PML for D2Q9 model**

We consider the classical acoustics problem :

$$\frac{\partial \rho}{\partial t} + \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0, \quad \frac{\partial j_x}{\partial t} + c_s \frac{\partial \rho}{\partial x} = 0, \quad \frac{\partial j_y}{\partial t} + c_s \frac{\partial \rho}{\partial y} = 0$$

and its PML version as proposed by Bérenger [1] :

$$\frac{\partial \rho_x}{\partial t} + \frac{\partial j_x}{\partial x} = 0, \quad \frac{\partial \rho_y}{\partial t} + \sigma \rho_y + \frac{\partial j_y}{\partial y} = 0, \quad \frac{\partial j_x}{\partial t} + c_s \frac{\partial (\rho_x + \rho_y)}{\partial x} = 0, \quad \frac{\partial j_y}{\partial t} + \sigma j_y + c_s \frac{\partial (\rho_x + \rho_y)}{\partial y} = 0$$

with a dissipation parameter $\sigma \geq 0$. In the domain of interest, we have the left hand side problem and we modelize it with the classical LBE D2Q9. In the PML we have the right hand side system of equations [2]. Here we consider the particular case where the PML interface is parallel to the x-axis. So the PML consists in splitting the equation of continuity in two equations. There is also an additional term with σ. To simulate this system of equations for the PML, we adapt the D2Q9 model with four conserved moments as there are four equations.

- **First test of the PML for LBE and difficulties**

First, to test this PML, we fix the different parameters (equilibrium values, ...) of LBE to have a stable algorithm [3] and the same sound speed c_s in the two domains. The numerical test shows that we have a reflected wave into the medium of interest. This reflexion is due to the difference of viscosity between the two media (domain of interest and PML). A Chapman-Enskog analysis of the PML model up to order two shows that the viscosity is not null neither isotropic.

So we analyse the transmission of an acoustic wave between two media wich have different speeds of sound and different viscosities. For the sake of simplicity we consider the simple D1Q3.

- **Analysis of conditions of transmission for D1Q3**

Let media 1 and 2 be two domains with sound velocity and viscosity c_1, ν_1, and c_2, ν_2, respectively. So, if we have an incident wave f_i of frequency ω with wave number k_1^+ in medium 1, there is a reflected wave f_r with wave number k_1^- and a transmitted one f_t with wave number k_2^- in medium 2. The theoretical coefficient of reflexion is defined by $r_{th} = \frac{J_r}{J_i} = \frac{k_2^+ - k_1^+}{k_1^+ + k_2^+}$. With the help of the hydrodynamic modes of D1Q3, and the study of the LBE algorithm in the two sites at left and right the interface, we find the coefficient of reflexion $r_{cal} = \frac{e^{ik_2^+} - e^{ik_1^+}}{e^{i(k_1^+ + k_2^+)} - 1}$. If we make an asymptotic development of k_1^+ and k_2^+ in ω, we find that $r_{th} = \frac{c_1 - c_2}{c_1 + c_2} + \frac{i\big(\nu_1 c_2^2 - \nu_2 c_1^2\big)}{c_1 c_2 (c_1 + c_2)^2}\, \omega + \mathrm{O}(\omega^2) = r_{cal} + \mathrm{O}(\omega^2)$. This confirms that the reflected wave in the present model of PML for LBE is due to the change of viscosity between the two media.

[1] J.-P. Bérenger, *Journal of Computational Physics*, Vol. **114**, n° 2, p. 185-200, 1994.
[2] F.Q. Hu, *Journal of Computational Physics*, Vol. **129**, n° 1, p. 201-219, 1996.
[3] P. Lallemand, L.S. Luo, *Physical Review E*, Vol. **61**, n° 6, p. 6546-6562, 2000.

*28 April 2006