COMPACT THIRD ORDER EXPANSION OF LATTICE BOLTZMANN SCHEMES

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The lattice Boltzmann scheme in his actual form has been developed with the contributions of Lallemand, Succi, d'Humières, Luo [1, 2, 3, 4] and many others. In order to derive the equivalent partial differential equations, a classical of the Chapman Enskog expansion is popular in the lattive Boltzmann community (see *e.g.* [4]). A main drawback of this approach is the fact that multiscale expansions are used without a clear mathematical signification of the various variables and functions. Independently of this framework, we have proposed in [5, 6] the Taylor expansion method to obtain formally equivalent partial differential equations. The infinitesimal variable is simply the time step (proportional to the space step with the acoustic scaling). This approach has been experimentaly validated in various contributions [7, 8]. A third order extension for fluid flow has been proposed in [9] and an efficient implementation up to fourth order accuracy is presented in [10].

In this contribution, we consider a regular lattice $\mathcal{L}$ composed by vertices x separated by distances that are simple expressions of the space step Δx. A discrete time t is supposed to be an integer multiple of a time step $\Delta t > 0$. A very general lattice Boltzmann scheme with q discrete velocities of the form

$$f_j(x, t+\Delta t) = f_j^*(x - v_j\,\Delta t, t)\,, 0 \le j < q\,.$$

The distribution f^* after relaxation is defined with moments m such that

$$m_k = \sum_j M_{k\ell}\, f_j\,.$$

The d'Humières matrix [3] M is invertible and we decompose the moments in the following way:

$$m \equiv \begin{pmatrix} W \\ Y \end{pmatrix}.$$

The conserved variables W are not modified after relaxation: $W^* = W$. The microscopic variables Y are changed in a nonlinear way by the relaxationprocess:

$$Y^* = Y + S\,(\Phi(W) - Y)\,.$$

The matrix S is invertible, and ofter chosen as diagonal. It is supposed to be fixed in the asymptotic process presented hereafter. The equilibrium values $Y^{\rm eq} = \Phi(W)$ are given smooth functions of the conserved variables. When $*^*$ is evaluated, we have simply

$$f^* = M^{-1}\, m^*\,.$$

With this general framework, we determine in this contribution an asymptotic system of partial differential equations for the conserved variables:

$$\partial_t W + \Gamma_1(W) + \Delta t\, \Gamma_2(W) + \Delta t^2\, \Gamma_3(W) = {\rm O}(\Delta t^3)$$

and an asymptotic expansion for the microscopic variables as a differential nonlinear function of the conserved variables:

$$Y = \Phi(W) + \Delta t\, \Phi_1(W) + \Delta t^2\, \Phi_2(W) + \mathrm{O}(\Delta t^3)\,.$$

The coefficients Γ_1, Φ_1, Γ_2, Φ_2 and Γ_3 of this expansion are recursively determined as a function of the data v_j, M, $\Phi(W)$ and S. We compare our new result with the particular third order expansion proposed in [9] and the linear approach presented in [10].

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