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BOUNDARY CONDITIONS IN REGULARIZED LATTICE-BOLTZMANN

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Abstract. The lattice-Boltzmann equation (LBE) can be considered as a projection of the Boltzmann equation onto a subspace Hq of the Hilbert space H that maps the velocity space onto the real numbers. The dimension q of Hq is dependent on the hydrodynamic problem it is wanted to solve, whether isothermal or non-isothermal, or if it involves non-ideality in single or multi-component systems [1] [2], [3]. Since subspace Hq is generated by a finite Hermitian basis, truncated at a given order, the solution of a LBE involve errors being affected by high-order moments that cannot be controlled with this approximation and considered to contribute to instability issues. Regularization is not a new concept in LBM and dates to the pioneer works of Ladd [4] in 1994 and showed to have improved stability properties by Latt & Chopard [5]. An improvement of the regularization method was proposed, in connection with kinetic projections [6] and it was demonstrated that solutions of the LB equations, with improved stability ranges, may be found, in a systematic way, based on increasingly order projections of the continuous Boltzmann equation onto subspaces generated by a finite set of Hermite polynomials. We considered a particular truncation, filtering the diffusive parts of high-order non-equilibrium moments that do not belong to the Hilbert subspace Hq, retaining. only their corresponding advective parts that fit into this representation. The decomposition of moments into diffusive and advective parts is based directly on general relations between Hermite polynomial tensors. The resulting regularization procedure led to recurrence relations where high-order non-equilibrium moments were expressed in terms of low-order ones [6]. The procedure is appealing in the sense that stability can be enhanced without local variation of transport parameters, e.g., the viscosity, or without tuning the simulation parameters based on embedded optimization steps.
In this work, LB regularization is extended to boundary conditions (BC). Dealing with boundary conditions was ever considered a puzzling question in LBM, especially, when a large set of lattice vectors is required for the description of a given physical problem. The most popular BC models are based on Ad-Hoc rules, [7], [8] and although these BC models were shown to be suitable for low-order LBE, their extension to high-order LBE was shown to be a very difficult problem and, at authors knowledge, never solved with satisfaction. In fact, the main question to be solved is how to deal with a problem when the number of unknowns (the particle populations coming from the outside part of the numerical domain) is greater than the number of equations we have at each boundary site. A new boundary condition model is here proposed. The main idea is that when we write both the equilibrium and non-equilibrium parts of the discrete populations f_i in terms of its equilibrium and non-equilibrium hydrodynamic moments, these moments replace the discrete populations as unknowns,

independently of the number of discrete velocities that are needed for solving a given problem. This idea was applied to the 2D [9] and 3D [10] lid-driven cavity flow problem and improved stability properties were demonstrated with respect to Zou & He [7] model for boundary conditions.

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