Immersed Boundary Method applied to the LBM

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1. Theory & equations

2. Academic test cases :
   a) 2D Oscillating cylinder in a laminar flow
   b) 3D translating sphere in a laminar flow
   c) 2D Obstructed channel in a laminar flow
   d) 3D Wall modeled LES oscillating cylinder in a turbulent flow

3. Industrial test case
   a) 3D rotating GMV (Groupe Moto Ventilateur) in a turbulent flow

4. Conclusion and Work in progress

Theory and equations

- Lattice-Boltzmann equation : $$f_i(x + c_i\Delta t, t + \Delta t) = f_i^{eq}(x,t) + \left(1 - \frac{1}{\tau}\right) f_i^{neq}(x,t) + \frac{1}{2} h_i(x,t)$$

- Expression of the external force g in the lattice space : $$h_i = \omega_i \left(1 - \frac{1}{2\tau}\right)\left[\frac{c_i - u}{c_s^2} + \frac{c_i . u}{c_s^4} c_i\right] . g$$

- Expression of the equilibrium function (HRR collision model) : $$f_i^{eq} = \omega_i \left[\rho + \frac{c_{i\alpha}\rho u_\alpha}{c_s^2} + \frac{a_{\alpha\beta}^{(2),eq} H_{i\alpha\beta}^{(2)}}{2c_s^4} + \frac{a_{\alpha\beta\gamma}^{(3),eq} H_{i\alpha\beta\gamma}^{(3)}}{6c_s^6}\right]$$

- Expression of the non equilibrium function (HRR collision model) : $$f_i^{neq} = \omega_i \left[\frac{a_{\alpha\beta}^{(2),neq} H_{i\alpha\beta}^{(2)}}{2c_s^4} + \frac{a_{\alpha\beta\gamma}^{(3),neq} H_{i\alpha\beta\gamma}^{(3)}}{6c_s^6}\right]$$

- Expression of the momentum with an external force g : $$\rho u = \sum_i c_i f_i^{eq} + \frac{g}{2}\Delta t$$

Theory and equations

Gsell et al, 2019,2021 Favier, 2016

- Update of the position of the lagrangian markers X_k and the velocity of the solid U^d.
- Algorithm of the LBM without solid on the Eulerian fluid nodes x_i
- Loop on the Lagrangian solid markers X_k :
  - Interpolation of ρ and u on the Lagrangian markers:

$$I[\rho](X_k) = \sum_{xi} \rho(x_i)\,\delta(x_i - X_k)\Delta S_i$$

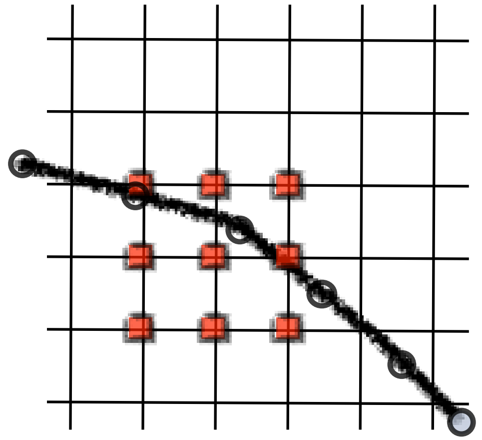
  - Computation of the IBM force :

$$G(X_k, t) = \frac{2}{\Delta t}\left(I[\rho]U^d(X_k, t) - I[\rho u^*]\right)$$

  - Spreading of the force on the surrounding eulerian fluid nodes :

$$g(x_i) = S[G](x_i) = \sum_{k} G(X_k)\,\delta(x_i - X_k)\Delta S_k$$

- Update of the velocity u and of the distribution function f_i



: Eulerian nodes X_k

: Lagrangian nodes x_i

$$\delta(r) = \begin{cases} \frac{1}{2d}\left(1 + \cos\left(\frac{\pi r}{d}\right)\right), & |r| \leq d, \\ 0, & |r| > d, \end{cases}$$

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3. Industrial test case

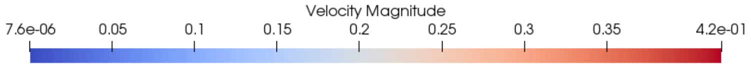
a) 3D rotating GMV (Groupe Moto Ventilateur) in a turbulent flow

4. Conclusion and Work in progress

2D laminar fixed case : cylinder at Re = 30

Classical method in ProLB

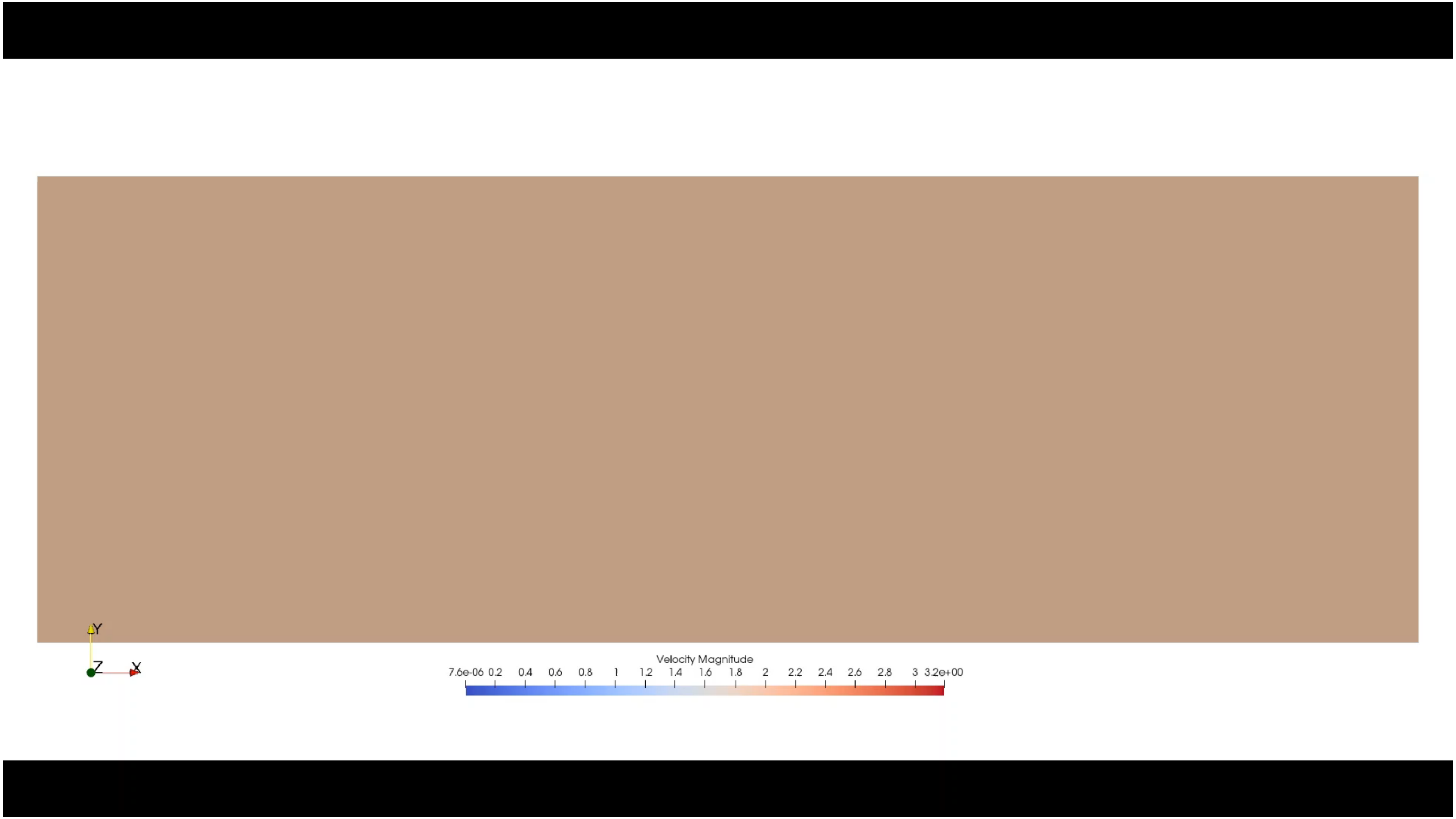
Immersed Boundary Method in ProLB



Mean drag coefficient on a cylinder for Re = 100

Study	$\overline{C_d}$
Braza, Chassaing & Ha Minh (1986)	1,28
Zhou, So & Lam (1999)	1,48
Shen, Chan & Lin (2009)	1,38
Bourguet & Jacono (2014)	1,32
Gsell & Favier (2019)	1,37
Present	1,42

2D laminar moving case : oscillating cylinder at Re = 185

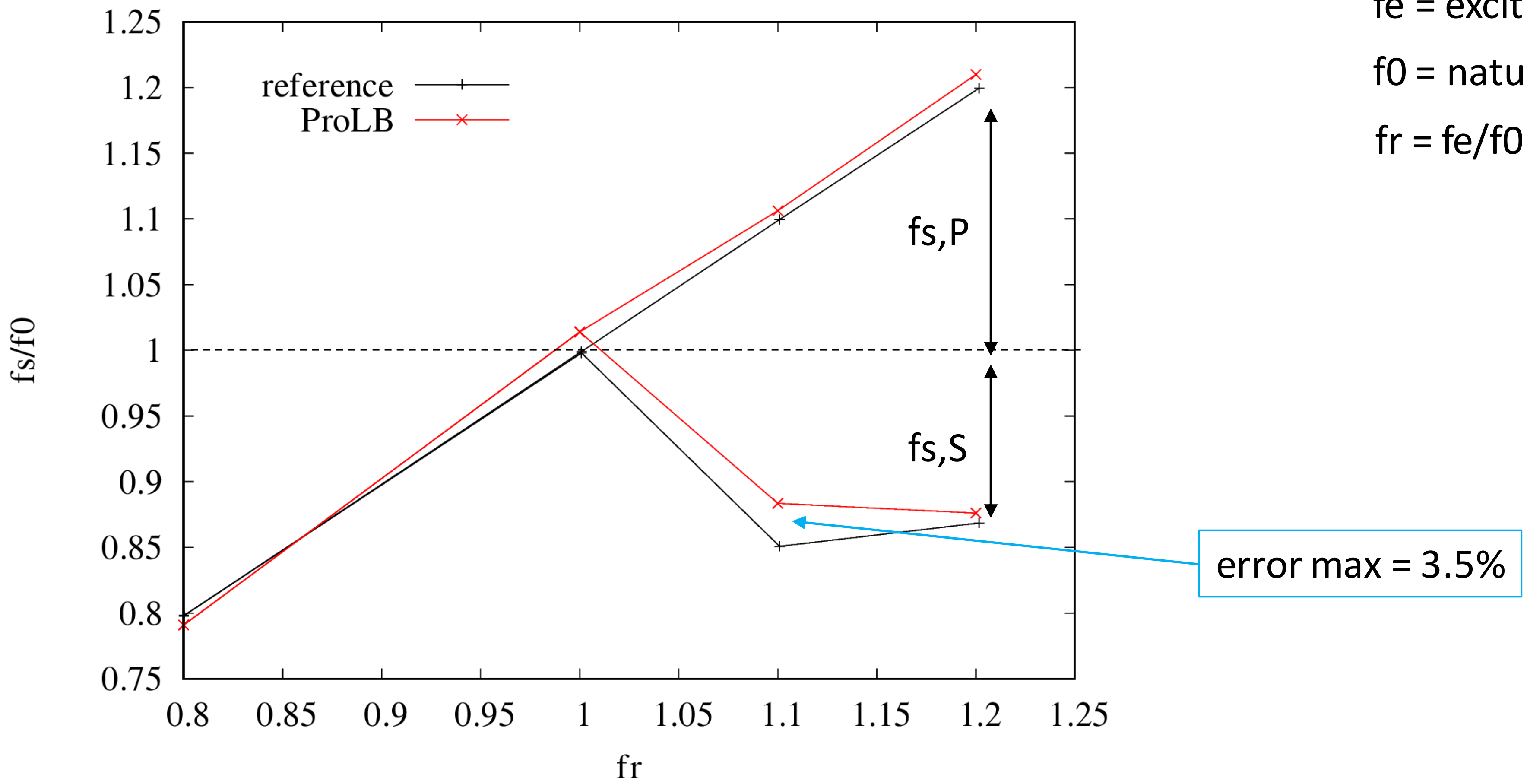


Ae = oscillating amplitude

fe = exciting frequency

f0 = natural shedding frequency for Re=185

fr = fe/f0

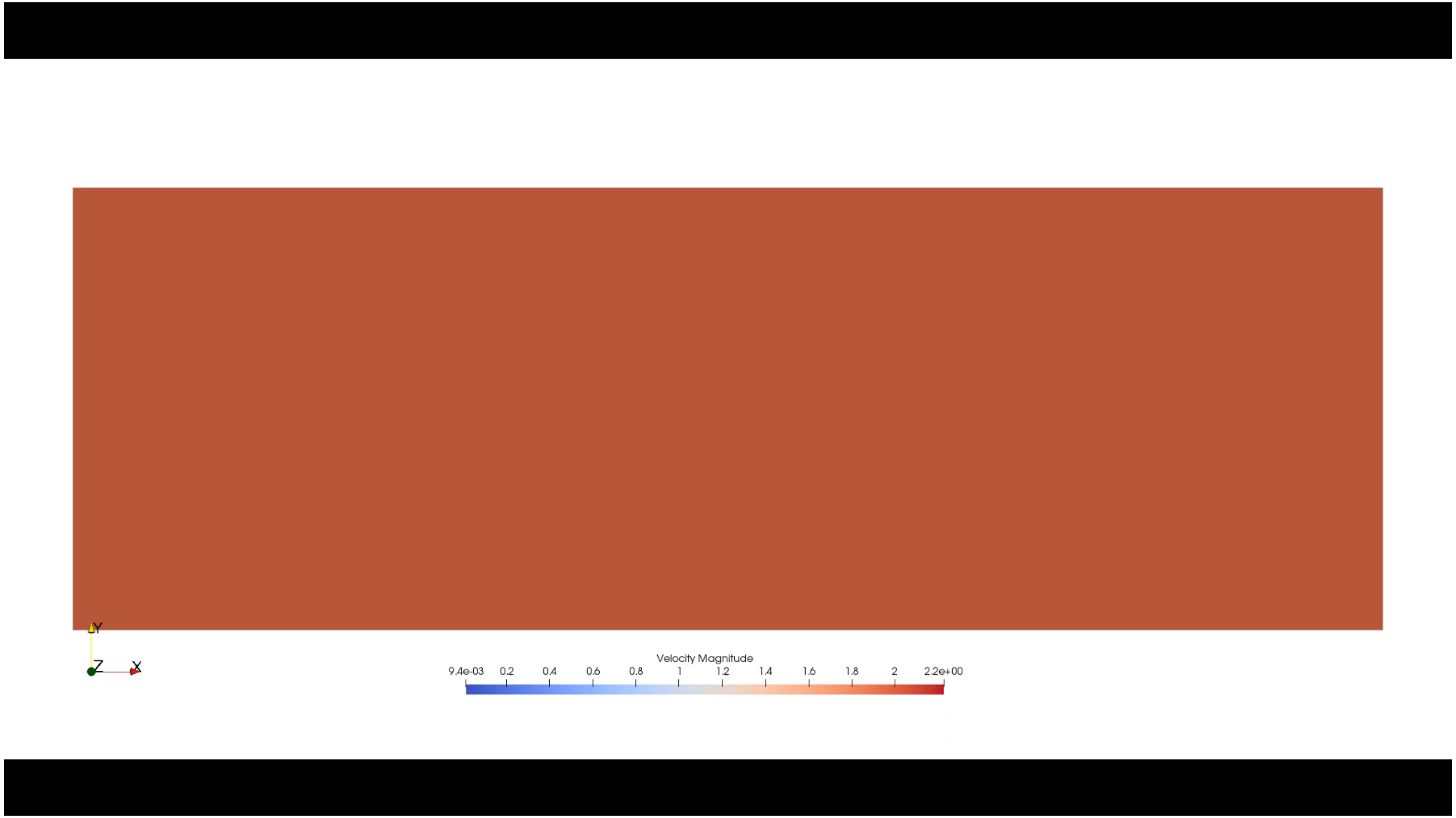


Normalized vortex shedding frequency by the natural shedding frequency f0 as a function of the fr, where fs,P and fs,S are the primary and secondary frequencies of fs, respectively

3D laminar moving sphere at Re = 185

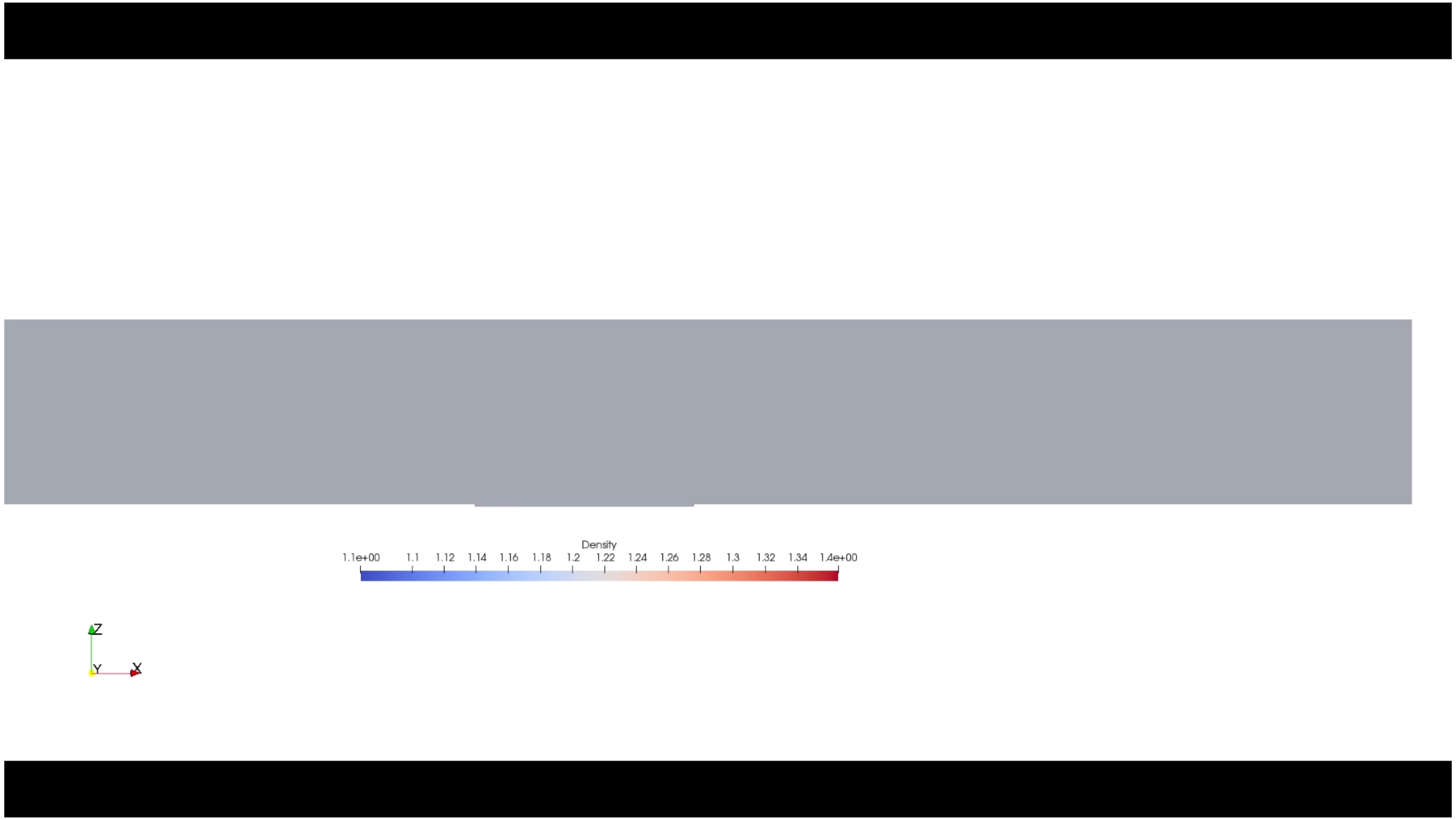
- Demonstration test case to show the industrial possibilities
- Translation imposed to the sphere
- The solver runs in parallel (22 processors) in 3D
- 3 meshing zones
- 2 155 949 nodes

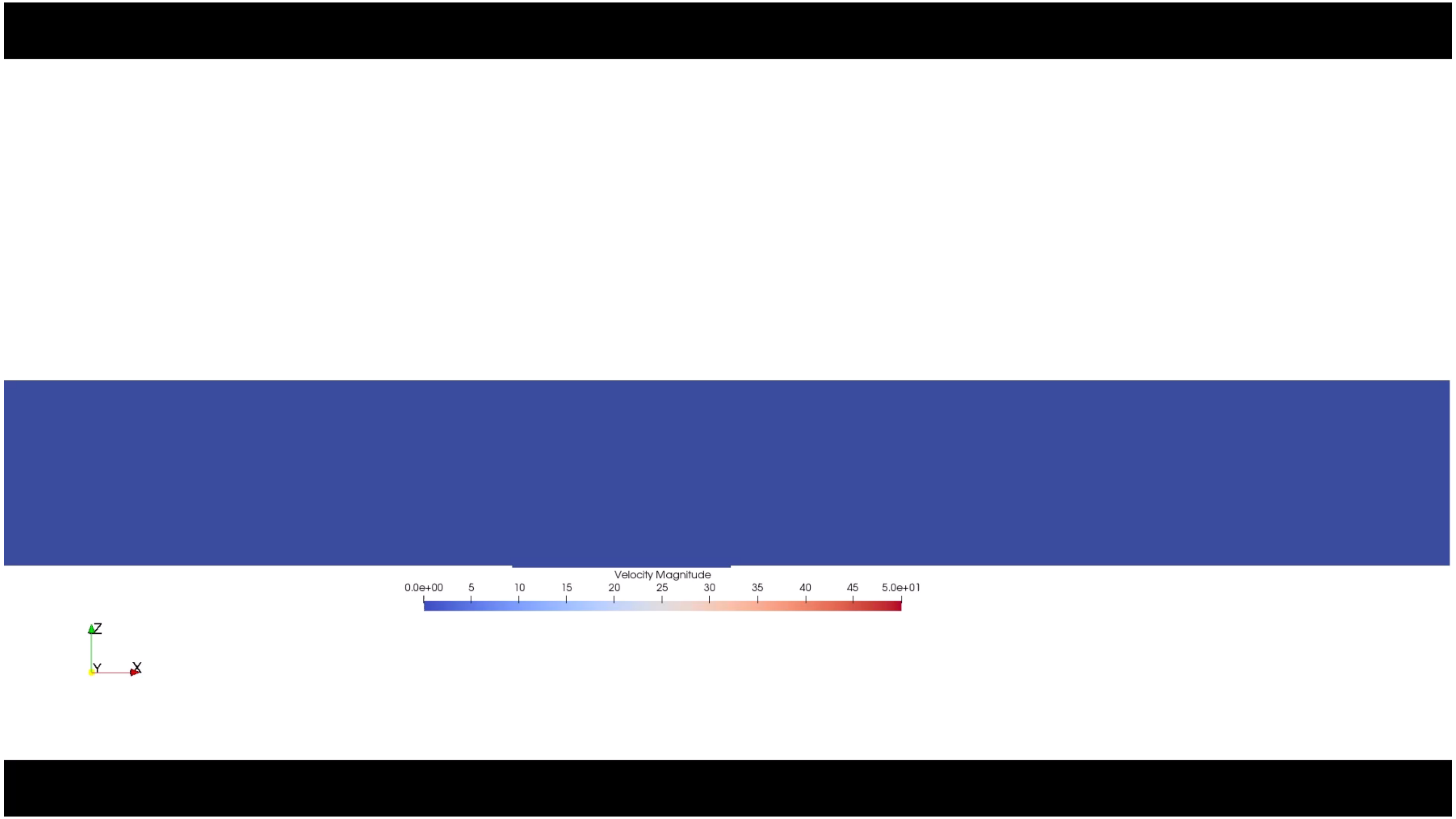
3D laminar moving case: sphere at Re = 185



Obstructed channel

- Inlet Reynolds number ≈ 4200
- A valve closes the channel and then opens it again
- What are the highlights :
  - Can the IBM support contact between 2 solids for some time ?
  - Is there some leakage across the valve ?




Velocity Magnitude
0.0e+00 5 10 15 20 25 30 35 40 45 5.0e+01
Z
Y
X

Theory

Mass at the left of the valve

Entrance flow rate

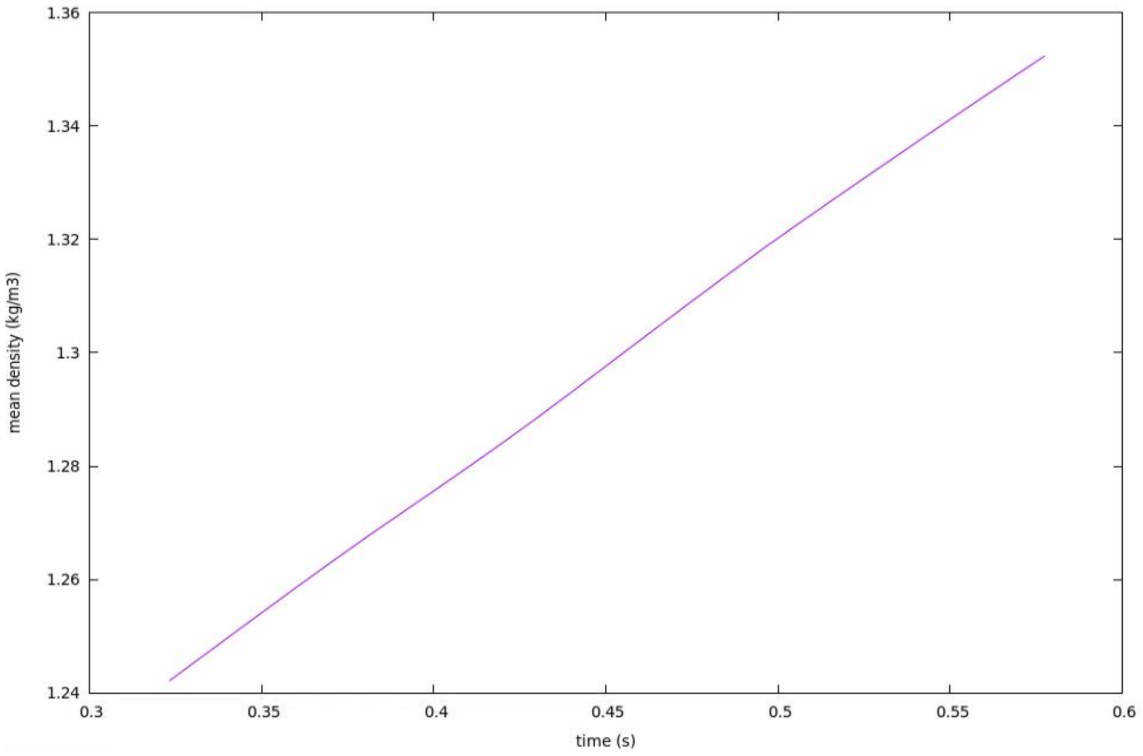
- $m(t) = m(t_0) + Q\ t$

t_0 = when the valve closes the channel

- $m(t) = \int_V \rho\, dV$

- $Q = \rho\ \|\overrightarrow{v_0}\|\ S$

Simulation



- Q = V * slope

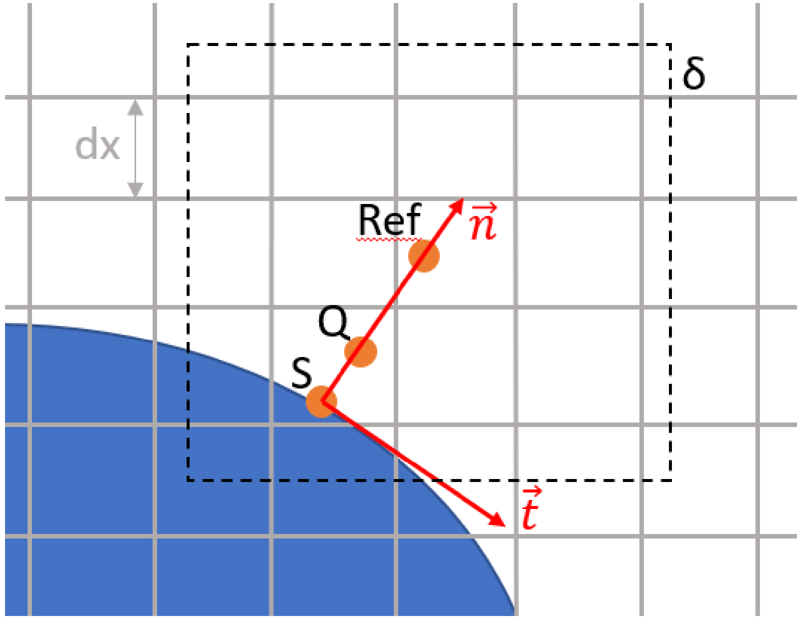
≈ 1 % error on Q due to leakage across the valve

Turbulent case: coupling of a turbulent wall law to the immersed boundary

Shi et al, 2019

- Set of **Lagrangian points near the wall** are introduced to compute the normal component of the IB force by reconstructing the normal component of the velocity

- The momentum equation is integrated along the wall-normal direction to link the **tangential component** of the IB force to the **wall shear stress predicted by the wall model**



Obtained with the wall law

$$
\begin{cases}
F_t(Q) \approx 2\dfrac{\tau_w}{dx} \\
F_n(Q) = \dfrac{0 - I_Q[u_n]}{dt}
\end{cases}
$$

Algorithm of the turbulent immersed boundary

By integrating the momentum equation along the normal direction to link the effective body force to the wall shear stress:

$$\int_0^{dx/2} g.e_\xi \, dx = \int_0^{dx/2} \left(\frac{\partial u}{\partial t} + u.\nabla u + \nabla p - \frac{1}{Re}\nabla^2 u\right).e_\xi \, dx$$

By integrating the viscous terms between the wall and Ref point:

$$G_\xi(Q) = \left(\frac{\partial(U.e_\xi)}{\partial t} + U.\nabla U.e_\xi + \frac{\partial P}{\partial \xi}\right) + \frac{\tau_w}{dx/2} - \frac{\tau_{dx/2}}{dx/2}$$

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For the wall-modeled LES of a turbulent flow, the right-hand side can be approximated by the dominant wall shear stress term as follows:

$$\begin{cases} G_\xi(Q) = 2\dfrac{\tau_w}{dx} \\ G_n(Q) = \dfrac{2}{\Delta t}\left(I[\rho]U_n^d - I[\rho u_n^*]\right) \end{cases}$$

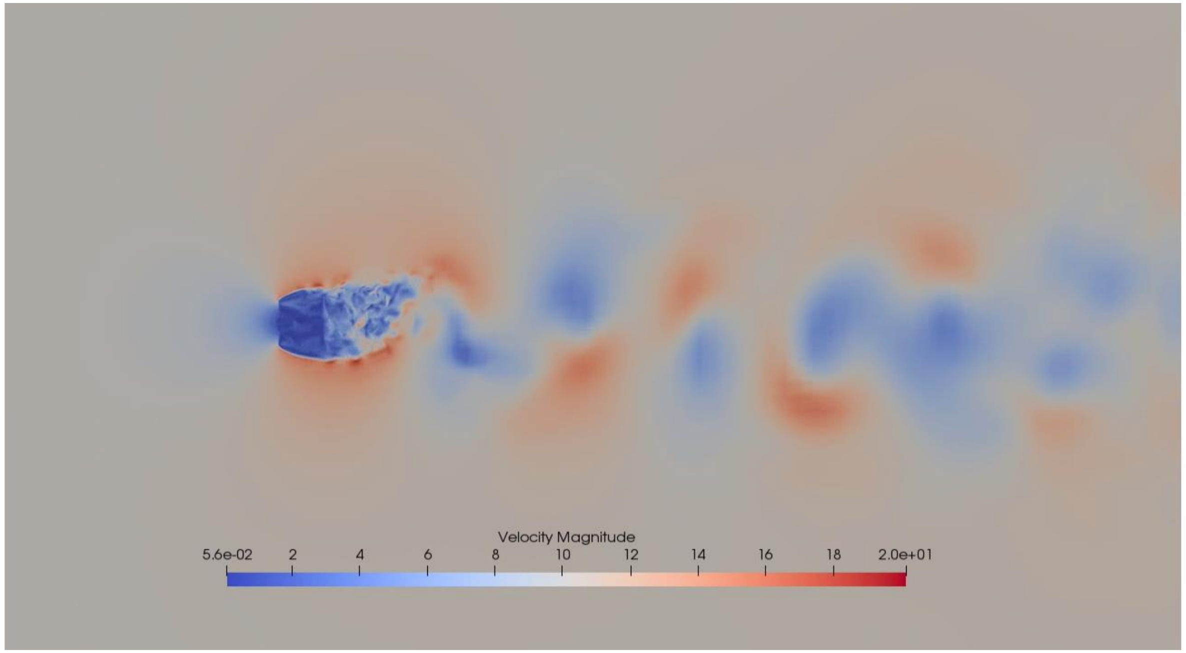
Algorithm of the turbulent immersed boundary

- Algorithm of the LBM without solid on the Eulerian fluid nodes x_i

- Loop on the Lagrangian solid triangles Δ_k
  - Computation of the normal and center of the triangle k , of point *Ref* at a distance 2.5 dx, and point *Q* at a distance dx/4
  - Interpolation of ρ and u on Ref : $I[u](Ref) = \sum_i u(x_i)\, \delta(x_i - Ref)\Delta S_i$
  - Calculation of the Ref tangential and normal velocity $u_{t,\mathrm{ref1}}$ and $u_{n,\mathrm{ref1}}$
  - Computation of the friction velocity u_τ with the power wall law : $u^+_{Ref} = \begin{cases} y^+_{Ref} & \text{if } y^+_{Ref} \le y^+_c \\ A(y^+_{Ref})^B & \text{if } y^+_{Ref} \ge y^+_c \end{cases}.$
  - Computation of the wall shear stress : $\tau_w = \rho u_\tau^2$
  - Computation of the immersed boundary force
  - Spreading of the force on the surrounding eulerian fluid nodes : $g(x_i) = \sum_k G(Q)\, \delta(x_i - Q_k)\Delta S_k$

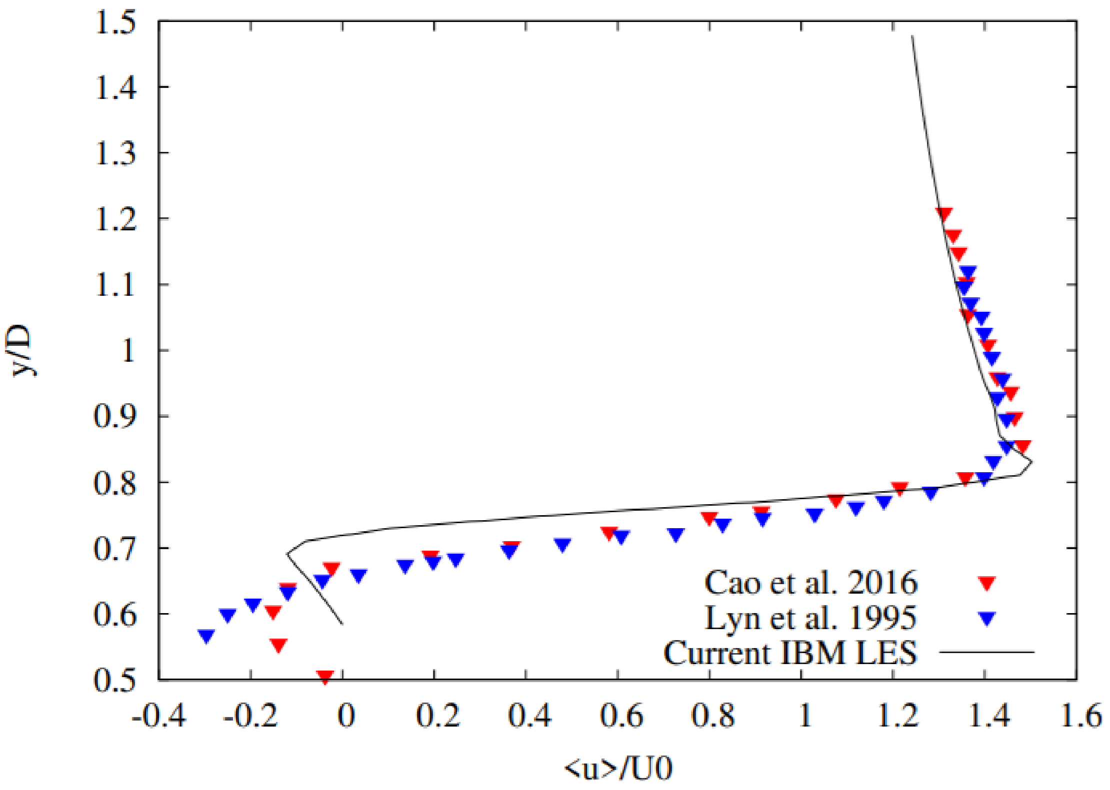
Large eddy simulation of a square cylinder at Re = 22000, Chen 2020

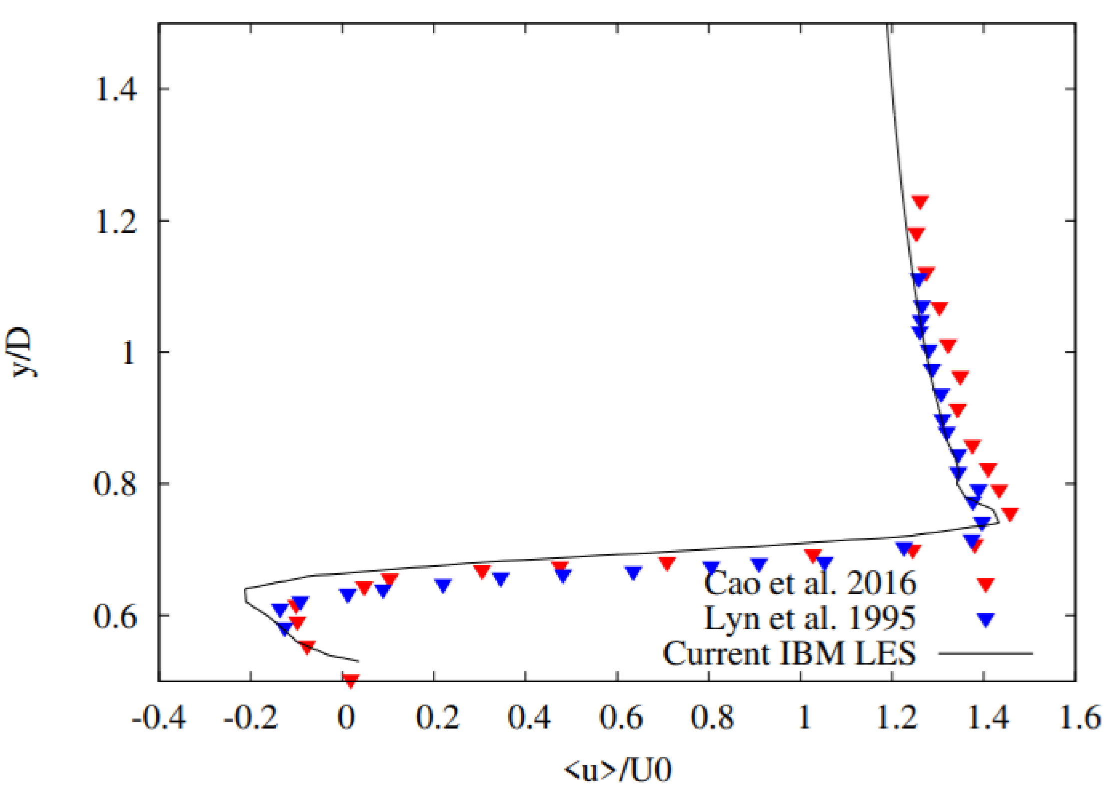
- Fixed Square cylinder
- Reynolds number = 22000
- Smagorinsky model with a power wall law
- 50 nodes on the diameter



Study	$\overline{C_d}$	St	$C_{d,RMS}$	$L_f(D)$
Lyn et al. (Experiment, 1995)	2.11	0.13	-	1.37
Minguez et al. (Experiment, 2011)	2.1	0.13	-	-
Chen et al. (LES, 2020)	2.246	0.135	0.14	1.1
Cao and Tamura (LES, 2016)	2.11-2.30	0.126-0.138	0.086–0.273	1.03–1.25
Present	2.09	0.14	0.14348	1.009

Large eddy simulation of a square cylinder at Re = 22000, Chen 2020

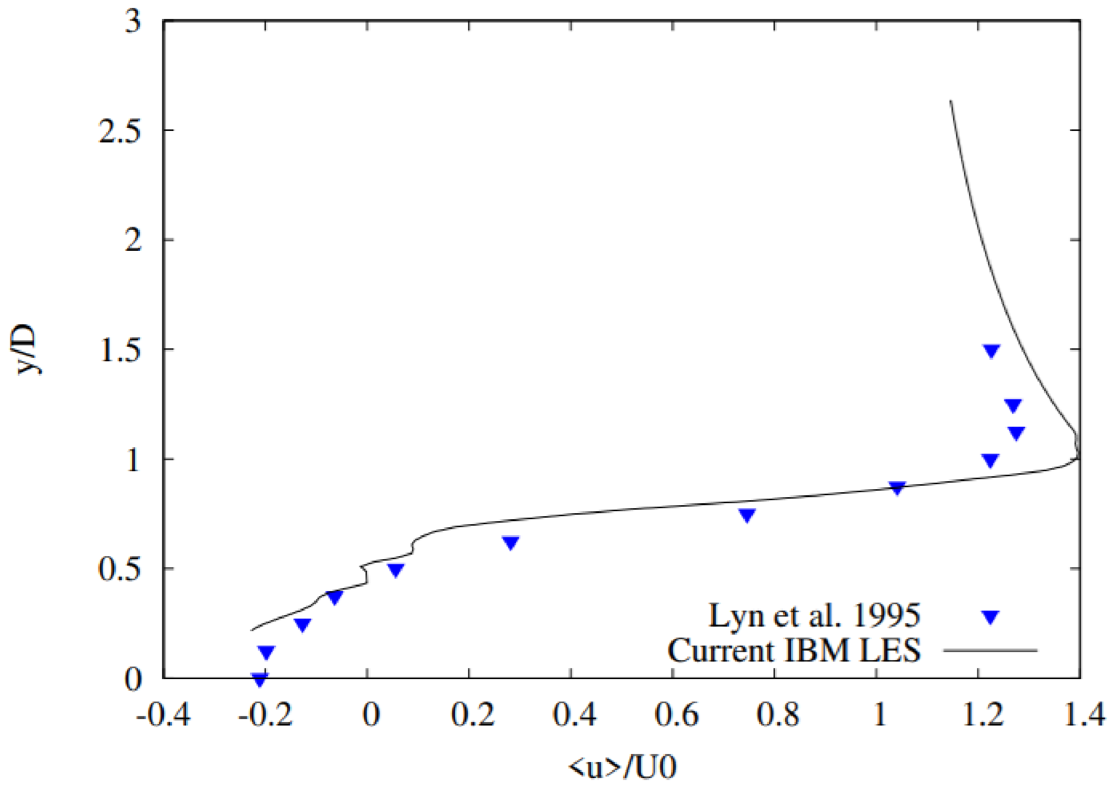




Velocity profile at the top of the cylinder in the **shear layer** at x/D = -0.25 (left) and x/D = 0 (right)
Comparison with experimental measurements (Lyn et al., 1995) and LES (Cao et al., 2016)

Large eddy simulation of a square cylinder at Re = 22000, Chen 2020

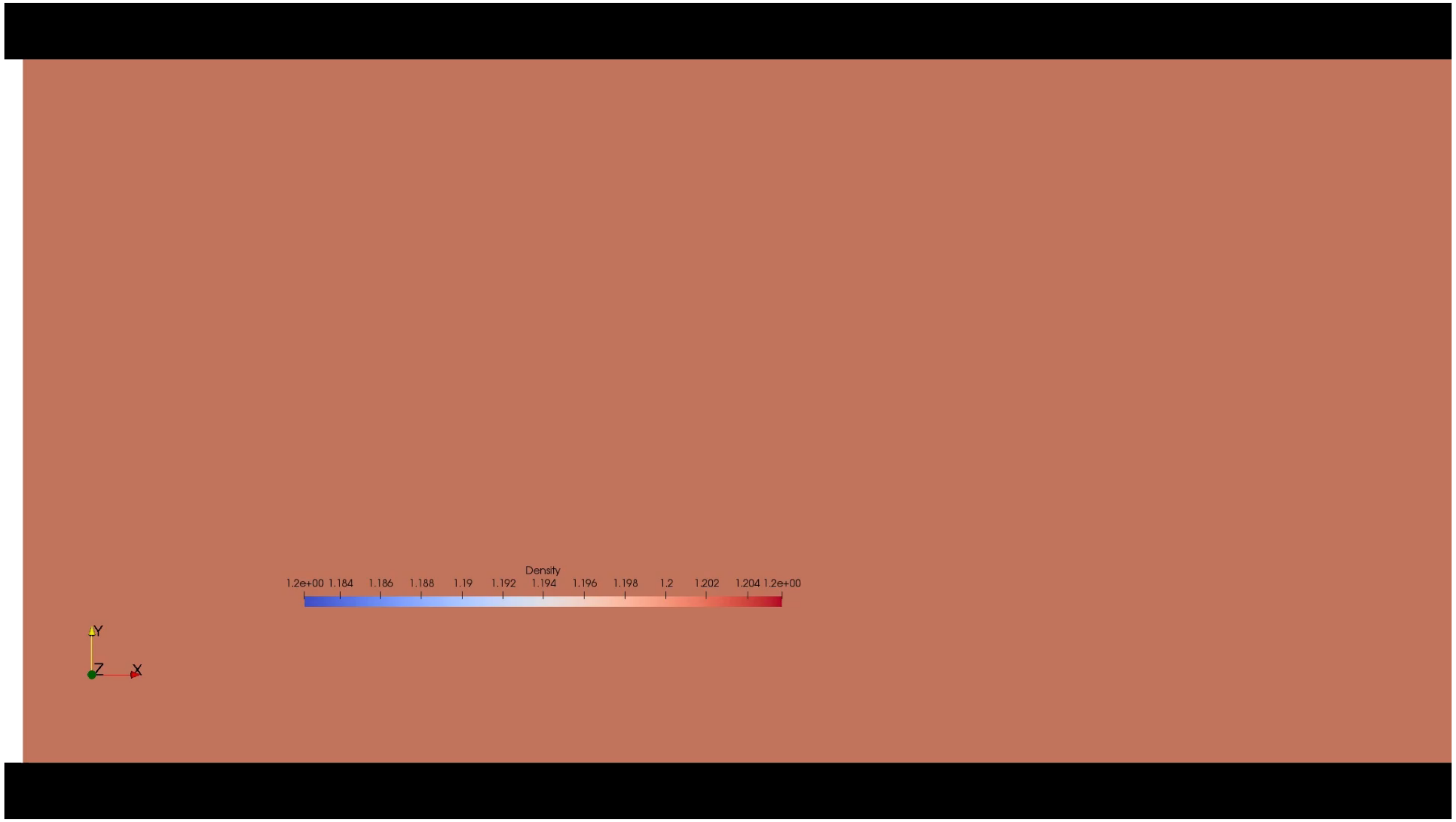
- Separated shear layer dynamics is very well captured on the top and bottom sides of the cylinder
- Shear layer evolution downstream the trailing edge corners is also well predicted



Velocity profile at the right of the cylinder in the **near wake** at x/D = 0.875
Comparison with experimental measurements (Lyn et al., 1995)

Large eddy simulation of an oscillating square cylinder at Re = 22000, Chen 2020

- Square cylinder oscillating at a low amplitude
- Reynolds number = 22000
- Smagorinsky model with a power wall law
- 50 nodes on the diameter



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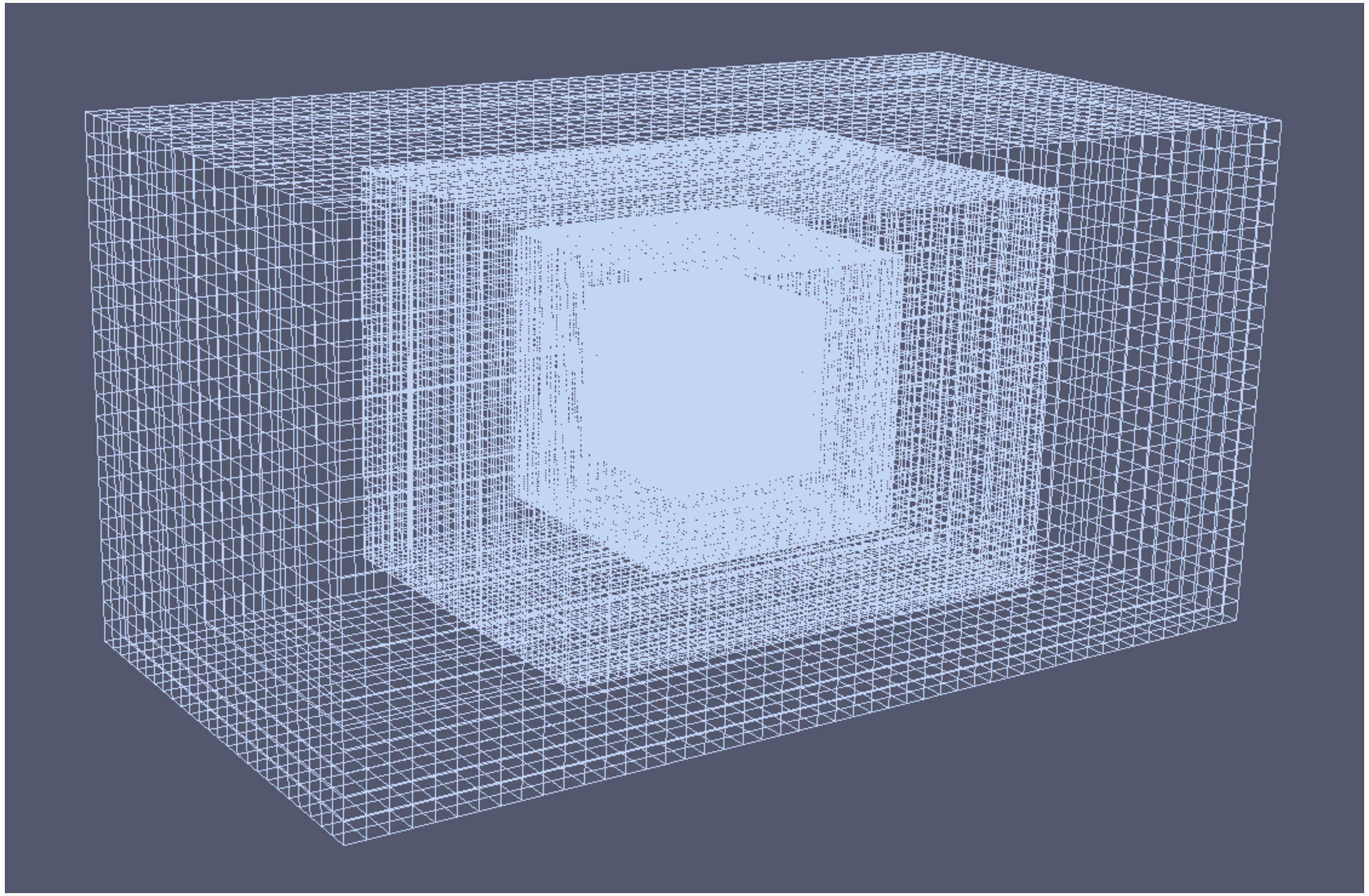
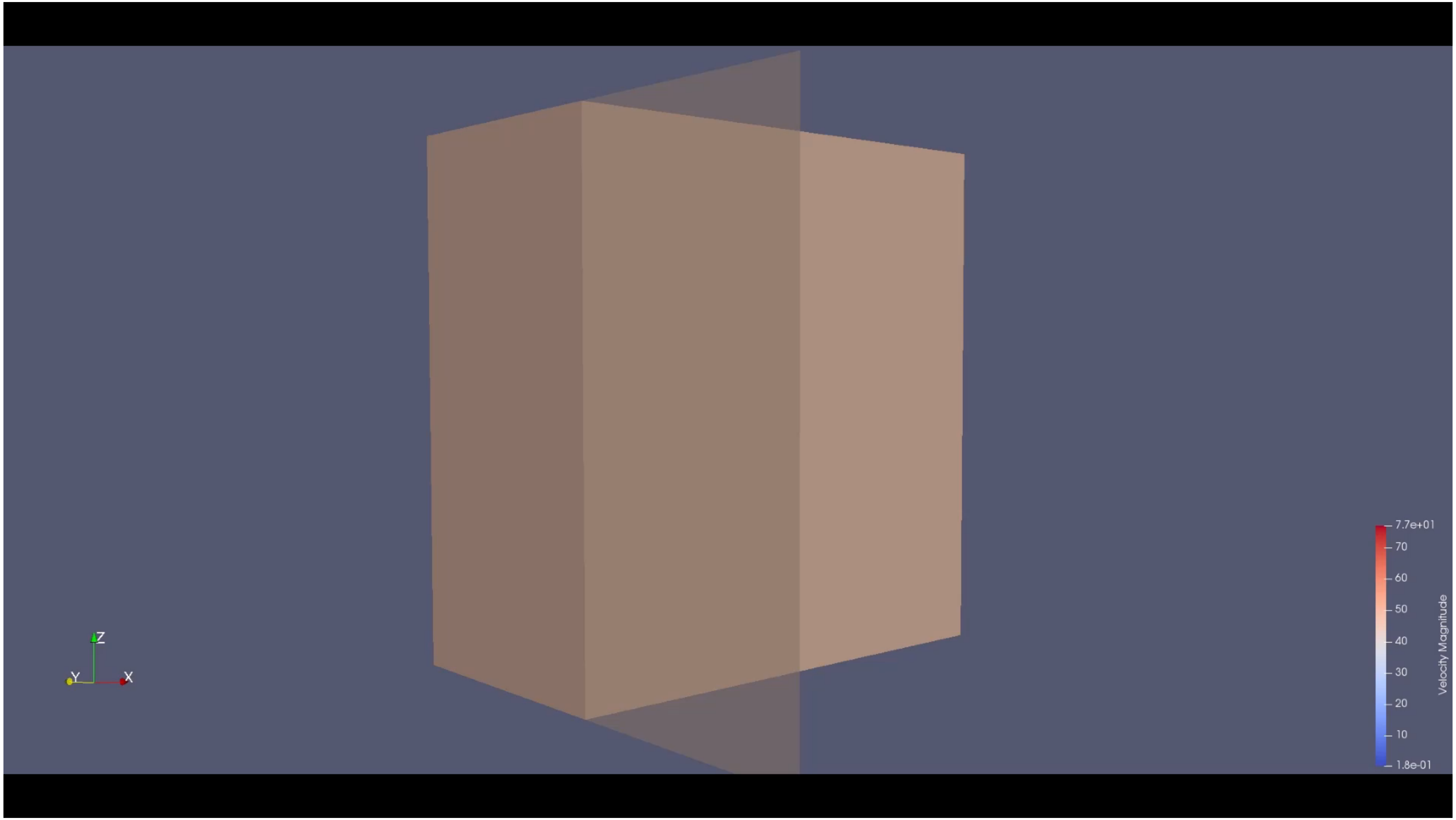
4. Conclusion and Work in progress

Data setup

- Inlet velocity = 45,6 m/s
- angular velocity = 0.22 rad/s
- LES turbulence model (SISM)
- 1.5 million nodes
- 640 processors





Z
Y
X
7.7e+01
70
60
50
40
30
20
10
1.8e-01
Velocity Magnitude

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Work in progress

- Fluid structure interaction
- Contact between 2 solids
- Reciprocity of the interpolating and spreading operators in the immersed boundary method

Reciprocity of the interpolating and spreading operators

Le principe est le suivant : en faisant un spreading de cette fonction et en interpolant le champ Eulérien obtenu par le spreading, on doit retrouver la fonction de départ. Pour chaque point Lagrangien $l = 1 \ldots N_s$, cela s'écrit :

$$\phi(\mathbf{q}_l) = \sum_{j \in D_{q_l}} \left(\sum_{k \in D_j} \phi(\mathbf{q}_k)\delta_h(\mathbf{x}_j - \mathbf{X}(\mathbf{q}_k))\epsilon(\mathbf{q}_k) \right) \delta_h(\mathbf{x}_j - \mathbf{X}(\mathbf{q}_l))\Delta x \Delta y \Delta z \tag{1.12}$$

On peut reformuler l'équation (1.12) sous la forme suivante :

$$\phi(\mathbf{q}_l) = \Delta x \Delta y \Delta z \sum_{k \in D_j} A_{kl}\epsilon(\mathbf{q}_k)\phi(\mathbf{q}_k) \tag{1.13}$$

avec :

$$A_{kl} = \sum_{j \in D_{q_l}} \delta_h(\mathbf{x}_j - \mathbf{X}(\mathbf{q}_k))\delta_h(\mathbf{x}_j - \mathbf{X}(\mathbf{q}_l)) \tag{1.14}$$

La matrice A définie par son terme générique A_{kl} à l'équation (1.14) est donc construite par le produit de chaque noyau d'interpolation k défini au $k$ième point Lagrangien avec les autres noyaux l définis sur les autres points Lagrangiens l. Ce produit est non nul uniquement si les supports des noyaux se recouvrent localement, c'est-à-dire lorsque l est voisin du noyau k, et donc la matrice A est à dominante diagonale. La condition pour que l'identité (1.13) soit vérifiée pour toute fonction ϕ revient au problème linéaire suivant :

$$A\epsilon = \mathbf{1} \tag{1.15}$$

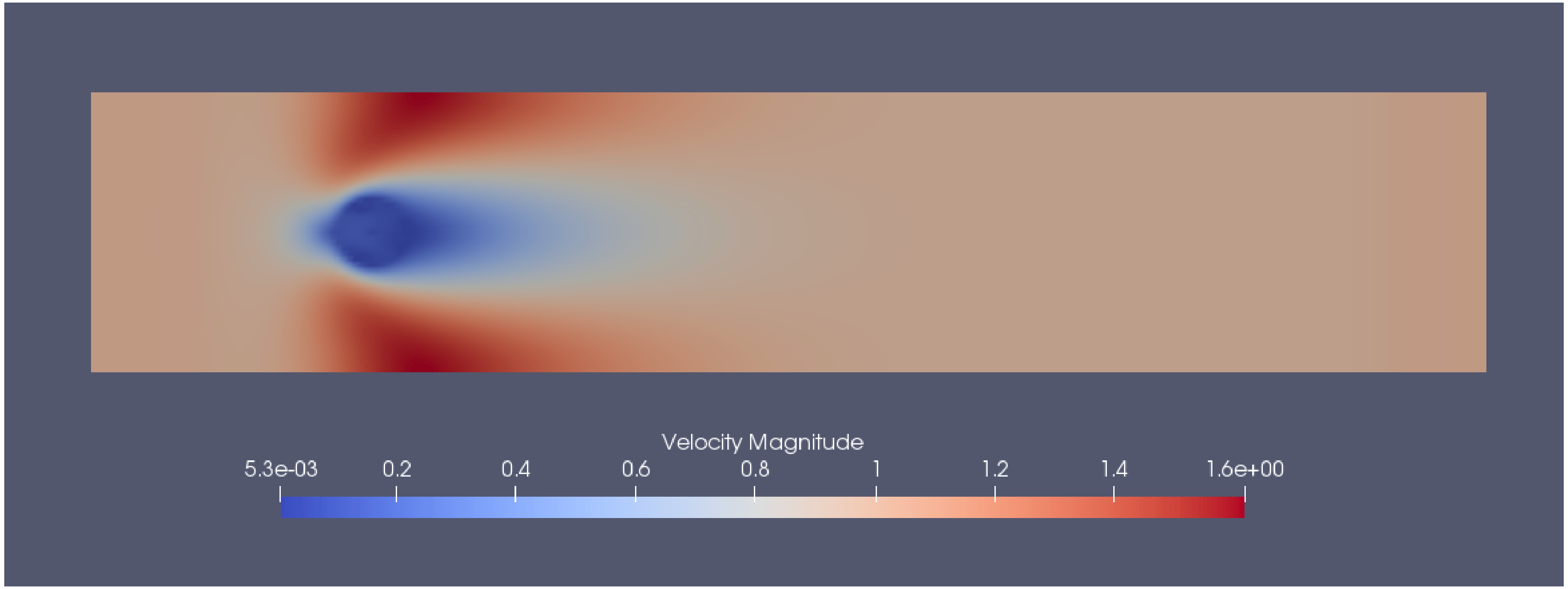
Linear system resolution

- Exact solution : **Matrix inversion**
  - ➢ Library used for the matrix inversion: LaPack (Linear Algebra Package)
  - ➢ Matrix construction : 2 intertwined loops on the lagrangian points
  - ➢ Computational time : **High extra CPU cost**

- Iterative solution**: BiConjugate gradient stabilized method**, without preconditioner
  - ➢ Directly implemented in ProLB
  - ➢ Similar matrix construction
  - ➢ Computational time : **Middle extra CPU cost**

- Analytical Approached solution
  - **No linear system resolution**
  - Minor modification in ProLB $I[S[G]] \approx \kappa G$ with $\kappa = \int \delta(x)^2 dx$
  - Identical computational time : **No extra CPU cost**

Calcul de l'erreur (RMSE) du flux de masse à la paroi du cylindre



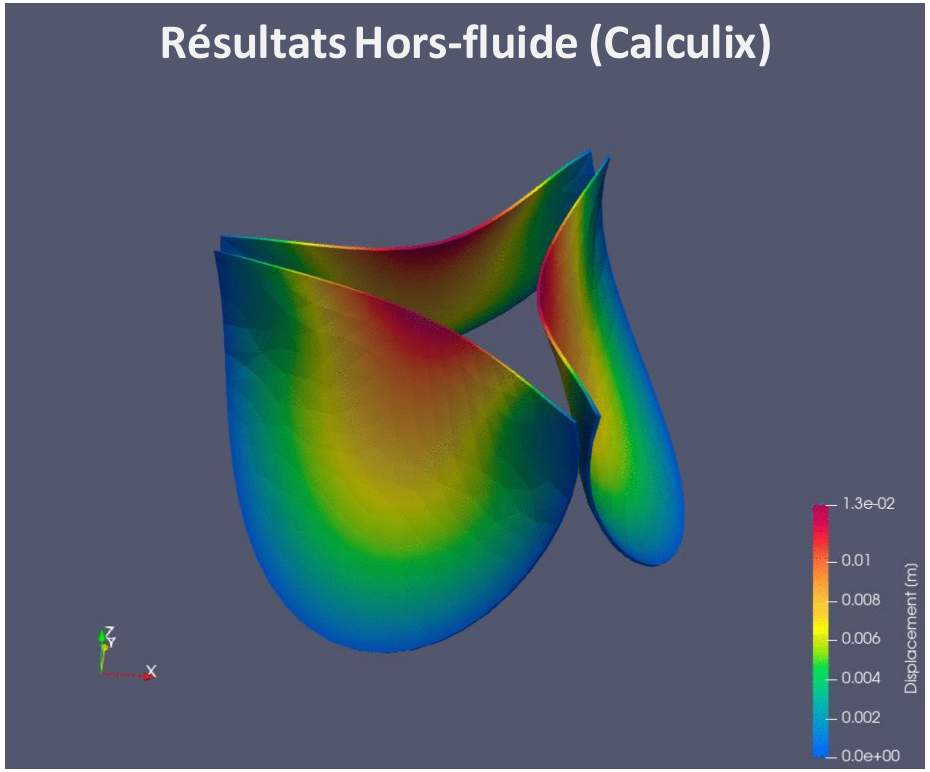
méthode	**Erreur ε**	**Temps de calcul**
Kappa	1^{e}-4	1
Bicgstab	5^{e}-5	1.6
Inverse matrice	1^{e}-5	2.25

$$\varepsilon = \sqrt{\frac{1}{N}\sum U^2}$$

Conclusion

- Development of a new coupling strategy to deal with moving boundaries in turbulent flows
- The power wall law is used for turbulent near wall modeling
- The immersed boundary method is coupled to the HRR LBM model and has proven to be accurate and robust on both academic and industrial test cases

Fluid-structure interaction in ProLB
Work in progress



Ouverture/fermeture des sigmoïdes sur un cycle cardiaque (pression imposée)