le cnam

Master Structural Mechanics and Coupled Systems

Applied Mathematics

Student presentation 2 (course 08)

- Generalized eigenvalues and eigenvectors

Let $n \geq 1$ an integer, K and M two a real symmetric positive definite matrices of order n. We consider the following Generalized eigenvalue problem: find $x \neq 0$ and a scalar λ such that $K\,x = \lambda\,M\,x$.

a) With the result of the previous exercice, prove that there exists a real symmetric positive definite matrix $M^{1/2}$ of order n such that $M = M^{1/2}.M^{1/2}$.

b) Transform the problem of finding $x \neq 0$ and a scalar λ such that $K\,x = \lambda\,M\,x$ into a classical spectrum problem associated to a real symmetric positive definite matrix A.

c) Explicit the matrix A in terms of the matrices introduced previously.

d) What are the orthogonality relations between the eigenvectors?

e) Solve the generalized eigenvalues problem when $K = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ and $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.

François Dubois, october 2022.