

Master of Structural Mechanics and Coupled Systems

Course on Applied Mathematics

Preliminary program

Lecture 1 Matrix calculus

- Definition of a matrix
- Equality and sum of two matrices; product of a number by a matrix
- Product of two matrices; non-commutativity of the product
- Transposition
- Identity matrix and zero divisor
- Determinant of a square matrix
- Determinant of the product of two square matrices
- Basic rules for the calculus of a determinant
- Fundamental theorem: a square matrix is invertible if and only if its determinant is not equal to zero

Lecture 2 Linear algebra

- The spaces \mathbb{R}^n and \mathbb{C}^n of line ($n \times 1$) or column ($1 \times n$) matrices
- Linear combination
- Vector spaces
- Vector space generated by a finite family of vectors
- Basis and dimension
- Linear map; kernel and image
- Criteria of bijectivity of an endomorphism
- Transformation matrix of a linear map relatively to a given basis
- Rank of a matrix; rank of a linear map
- Matrix formula for the output of a vector with a linear map
- Bijective linear map and invertibility of the associated matrix
- Change of basis; similar matrices
- Solving linear systems; Gaussian elimination in practice

Lecture 3 Eigenvalues and eigenvectors

- Fundamental theorem for the invertibility of a square matrix
- Determinant of a linear map
- Calculus of eigenvalues
- Characteristic polynomial
- Calculus of an eigenvector
- Diagonalization
- Linear operators that are not diagonalizable
- Polynomial of endomorphisms
- Theorem of Cayley and Hamilton
- Triangularization of a complex matrix

Lecture 4 Autoadjoint operators

- Symmetric matrices
- Scalar products in \mathbb{R}^n
- Symmetry criterion of a real square matrix
- Orthogonal subspaces
- Orthonormal basis
- Diagonalization of real symmetric matrices
- Basic optimization of linear quadratic functions
- Orthogonal matrices

Lecture 5 Linear differential systems

- A fundamental differential equation
- Linear differential equation with a right hand side
- Duhamel formula
- Cook book for particular solutions
- When the right hand side is solution of the homogeneous equation
- Structure of the space of solutions of a linear differential system
- Linear differential system associated to a diagonalizable matrix
- Differential equations of second order
- Dissipative harmonic oscillator

Lecture 6 Length, normal and curvature

- Distance between two points
- Oriented euclidian vector space
- Examples of plane and space curves
- Velocity
- A general formula for the length of a curve
- Curvilinear abscissa
- Unit tangent and normal vectors
- Curvature
- Frénet-Serret formulas for the TNB frame

Lecture 7 Curvilinear integral

- Definition
- Circulation of a vector field along a curve
- Differential calculus for functions of two and three variables
- Differential forms for two space variables
- Flux of a vector field for two space dimensions
- Change of tangent and normal vectors through a change of coordinates

Lecture 8 Double integral

- Fundamental properties
- Integral of a real combination of indicator functions (“simple functions”)
- Integral of a continuous function
- Theorems of Tonelli and Fubini
- Change of variables
- Integration by parts and Green formula
- Integral of the divergence of a vector field
- Green-Riemann formula
- Calculus of areas of plane surfaces

Lecture 9 Surface integral

- Surface of a parallelogram
- Cross product of two vectors

- Parametrization of a local surface
- Tangent plane
- Normal vector
- Approximated scale surface
- Area of a portion of a surface
- Flux of a vector field for three space dimensions
- Stokes theorem

Lecture 10 Triple integral

- Fundamental properties
- Integral of simple and continuous functions
- Boundary of a three-dimensional regular domain
- External normal
- Theorems of Tonelli and Fubini
- Change of variables; jacobians
- Integration by parts
- Integral of the divergence of a vector field

Lecture 11 Vector analysis

- Third order antisymmetric tensor
- Mixed product and cross product
- Gradient, divergence and curl operators
- Laplace operator
- Integration by parts the flux of a curl

Lecture 12 Supplements