Erratum: Deconvolution with unknown noise distribution is possible for multivariate signals

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The erratum offers another way to obtain an inequality similar to that given in Proposition A.2 in [Gassiat et al., 2022], since an error has been found in its proof.

In this note we use the setting and the notations introduced in [Gassiat et al., 2022].
The proof of Proposition A.2 uses a study of the linear part of $M_\star$ with a linear transformation A with properties described in Lemma I.1 of Appendix I in the supplementary material of [Gassiat et al., 2022]. In particular, this Lemma entails that A restricted to the polynomials of degree at most m is injective (lower triangular with diagonal coefficients equal to -1), with explicit inverse, thus allowing the control of its lowest singular value. The problem lies in point iv) of this Lemma: A is actually not injective. Its diagonal entries with coordinates $((i_1, 0), (i_1, 0))$ and $((0, i_2), (0, i_2))$ for $i_1, i_2 \geqslant 1$ are zero, and its entry with coordinate $((0, 0), (0, 0))$ is +1.

Consider now the following assumption. Let $\delta \in (0, 1)$.

H(δ): There exists $\nu_0 > 0$ such that for all $\nu \in (0, \nu_0]$, there exists $c(\nu, \delta) > 0$ such that for all $R^\star$ such that $\Phi_{R^\star} \in \mathcal{H} \cap \Upsilon_{\kappa,S}$,

$$\forall \phi \in \mathcal{H} \cap \Upsilon_{\kappa,S} \quad \text{s.t.} \quad \phi \neq \Phi_{R^\star}, \quad \frac{\left\| \dfrac{\phi}{\Phi_{R^\star}} - \dfrac{\phi^{(1)}}{\Phi_{R^\star}^{(1)}} - \dfrac{\phi^{(2)}}{\Phi_{R^\star}^{(2)}} + 1 \right\|_{2,\nu}}{\max\left(\left\| \dfrac{\phi^{(1)}}{\Phi_{R^\star}^{(1)}} - 1 \right\|_{2,\nu}, \left\| \dfrac{\phi^{(2)}}{\Phi_{R^\star}^{(2)}} - 1 \right\|_{2,\nu} \right)^{1+\delta}} \geqslant c(\nu, \delta)\,. \tag{1}$$

Proposition 1. *Assume* **H**(δ) *holds. There exists* $\overline{\nu} > 0$ *depending only on* S *and* ρ *such that for all* $\nu \leqslant \overline{\nu}$, *there exists a constant* $\widetilde{c} > 0$ *depending only on* c_ν *and* $c(\nu, \delta)$ *such that for all* $R^\star$ *such that* $\Phi_{R^\star} \in \mathcal{H} \cap \Upsilon_{\kappa,S}$, *all* $Q^\star \in \mathbf{Q}(\nu, c_\nu, c_Q)$ *for some* $c_Q \in (0, \infty]$, *for all* $\phi \in \mathcal{H} \cap \Upsilon_{\kappa,S}$, *as soon as*

$$\left\| \frac{\phi}{\Phi_{R^\star}} - \frac{\phi^{(1)}}{\Phi_{R^\star}^{(1)}} - \frac{\phi^{(2)}}{\Phi_{R^\star}^{(2)}} + 1 \right\|_{2,\nu} \leqslant \min\left\{ \left(c(\nu,\delta)^2 2^{-(1+\delta)} (2\nu)^{d(1+\delta)} \right)^{\frac{1}{1-\delta}} ; 1 \right\}, \tag{2}$$

it holds

$$M_\star(\phi;\nu) \geqslant \widetilde{c}\|\phi - \Phi_{R^\star}\|_{2,\nu}^{2(1+\delta)}. \tag{3}$$

In [Gassiat et al., 2022], $\mathcal{H}$ is chosen as a closed subset of $\mathbf{L}^2(B^d_{\nu_{\text{est}}})$ such that all elements of $\mathcal{H}$ satisfy **H2**. In the choice of $\mathcal{H}$ we may require that **H(**δ**)** holds, since this choice comes from the prior modeling that allows to fix **H2**, see examples in Section 2 of [Gassiat et al., 2022]. If we add **H(**δ**)** in the choice of $\mathcal{H}$, then Theorem 3.2 and Theorem 3.3 follow from the arguments developed in [Gassiat et al., 2022] with no modification.

Notice that the uniform consistency of the estimator (Section A.1 in [Gassiat et al., 2022]) holds without any change, that is without assuming **H(**δ**)** which is only used to get rates.

Let us now prove Proposition 1. First, notice that for all $\nu > 0$, for all $\phi \in \Upsilon_{\rho,S}$, $|\phi(t)| \leqslant \sup_{u:\|u\|\leqslant\|t\|} \|\phi'(u)\|\|t\|$, where $\phi'(u)$ denotes the gradient of ϕ at u (recall that ϕ is multivariate analytic), so that it is possible to choose $\overline{\nu}$ depending only on S and ρ such that for all $\phi \in \Upsilon_{\rho,S}$, all $\nu \leqslant \overline{\nu}$, all $t \in B^d_\nu$, $|\phi(t)| \geqslant 1/2$. Let now $R^\star$ be such that $\Phi_{R^\star} \in \mathcal{H} \cap \Upsilon_{\kappa,S}$, and let $Q^\star \in \mathbf{Q}(\nu, c_\nu, c_Q)$ for some $c_Q \in (0,\infty]$. Let ϕ be any function in $\mathcal{H} \cap \Upsilon_{\kappa,S}$ such that (2) holds. Denote

$$g_1 = \frac{\phi^{(1)}}{\Phi^{(1)}_{R^\star}} - 1, \qquad g_2 = \frac{\phi^{(2)}}{\Phi^{(2)}_{R^\star}} - 1 \quad \text{and} \quad G = \frac{\phi}{\Phi_{R^\star}} - 1 - g_1 - g_2\,.$$

Then, $M_\star$ rewrites

$$M_\star(\phi;\nu) = \|\Phi_{R^\star}\Phi^{(1)}_{R^\star}\Phi^{(2)}_{R^\star}(G - g_1 g_2)\Phi^{(1)}_{Q^\star}\Phi^{(2)}_{Q^\star}\|^2_{2,\nu}\,, \tag{4}$$

and we get $M_\star(\phi;\nu) \geqslant \frac{c_\nu^4}{2^6}\|G - g_1 g_2\|^2_{2,\nu}$. Using **H(**$\delta$**)** and (2), we get

$$\|g_1\|_{2,\nu}\|g_2\|_{2,\nu} \leqslant \left(\frac{\|G\|_{2,\nu}}{c(\nu,\delta)}\right)^{2/(1+\delta)} \leqslant \frac{\|G\|_{2,\nu}}{2}(2\nu)^d$$

(recall $0 < \delta < 1$). We then get

$$\begin{aligned}
M_\star(\phi;\nu) &\geqslant \frac{c_\nu^4}{2^6}(\|G\|_{2,\nu} - \|g_1 g_2\|_{2,\nu})^2\\
&= \frac{c_\nu^4}{2^6}(\|G\|_{2,\nu} - (2\nu)^{-d}\|g_1\|_{2,\nu}\|g_2\|_{2,\nu})^2\\
&\geqslant \frac{c_\nu^4}{2^8}\|G\|^2_{2,\nu}.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\|\phi - \Phi_{R^\star}\|_{2,\nu} &= \|G + g_1 + g_2\|_{2,\nu}\\
&\leqslant \|G\|_{2,\nu} + \|g_1\|_{2,\nu} + \|g_2\|_{2,\nu}\\
&\leqslant \|G\|_{2,\nu} + 2(c(\nu,\delta)^{-1}\|G\|_{2,\nu})^{1/(1+\delta)}\\
&\leqslant (1 + 2c(\nu,\delta)^{-1/(1+\delta)})\|G\|^{1/(1+\delta)}_{2,\nu}
\end{aligned}$$

since $\|G\|_{2,\nu} \leqslant 1$. Hence, we may conclude

$$M_\star(\phi;\nu) \geqslant \frac{c_\nu^4}{2^8(1 + 2c(\nu,\delta)^{-1/(1+\delta)})^{2(1+\delta)}}\|\phi - \Phi_{R^\star}\|^{2(1+\delta)}_{2,\nu}\,.$$

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References

[Gassiat et al., 2022] Gassiat, E., Le Corff, S., and Lehéricy, L. (2022). Deconvolution with unknown noise distribution is possible for multivariate signals. *Ann. Statist.*, 50(1):303–323.