Errata and Addenda for *Compact Matrix Quantum Groups and their Combinatorics*

Errata

- p. 176: one should read $\epsilon_{i+1} = (-1)^{n_{i+1}+1}\epsilon_i$ instead of $\epsilon_{i+1} = (-1)^{n_i+1}\epsilon_i$.
- p. 185: in Remark 7.3, one should read $G(s, 1, N)$ instead of $G(s, s, N)$.
- p. 215 : in the first step of the proof of Theorem 7.30, the first equation should be

$$\Phi_{ik} = (\nu_i \otimes \nu_k) \circ \Delta \circ \nu_i^{-1} : \nu_i(\mathcal{O}(\mathbb{G})) \to \mathcal{H} \otimes \mathcal{H} \to \mathcal{A} \otimes \mathcal{A}.$$

Addendum

On p. 7, following Definition 1.6 is a Remark (1.7) claiming that in the definition of $\mathcal{A}_s(N)$, the condition that the projections are pairwise orthogonal on the rows and columns does not follow from the other assumptions. However, that statement is not proven there and it seems that it does not appear in that form in the literature. We therefore fill the gap, thanks to argument pointed to us by J.P. McCarthy. This relies on a result from [**BES94**].

Proposition. Let $N \geqslant 5$. Then, there exists a $*$-algebra A_N generated by N self-adjoint projections $p_1, \cdots, p_N$ such that

$$\sum_{i=1}^{N} p_i = 1$$

but $p_1 p_5 \neq 0$.

Proof Note that if the result holds for $N = 5$, then for any $N > 5$ one may set $\mathcal{A}_N = \mathcal{A}_5$ and $p_i = 0$ for $i > 5$, so that we only have to prove the statement for $N = 5$. Let V be the vector space of all complex-valued functions on $\mathbf{R}$, and let define the following operators on V :

$$\begin{aligned} q_1(f) &: x \mapsto x\,[f(x) + f(1-x)] \\ q_2(f) &: x \mapsto x\,[f(x) - f(1-x)] \\ q_3(f) &: x \mapsto x\,[-f(x) + f(1-x)] \\ q_4(f) &: x \mapsto x\,[-f(x) - f(1-x)] \end{aligned}$$

An straightforward computation shows that $q_i^2 = q_i$ for all $1 \leqslant i \leqslant 4$, while

$$\sum_{i=1}^{4} q_i = 0.$$

As a consequence, setting $q_5 = 1$ yields a family of projections adding up to the identity, but such that $q_1 q_5 = q_5 \neq 0$.

We are not done however, since there is not involution on our algebra. To produce one, we will work at the universal level. Let $\mathcal{A}$ be the universal complex algebra generated by idempotent elements $p_1, \cdots, p_5$ such that

$$\sum_{i=1}^{5} p_i = 1.$$

By the universality, there is an algebra homomorphism $\pi : \mathcal{A} \to \mathcal{L}(V)$ sending p_i to q_i, so that in particular $p_1 p_5 \neq 0$. We now claim that $\mathcal{A}$ can be endowed with an involution making the generators self-adjoint. To see why, let $\mathcal{B}$ the opposite algebra of $\mathcal{A}$ with the conjugate complex structure. It is generated by five idempotents adding up to 1, hence by the universal property there is a $*$-homomorphism $\varphi : \mathcal{A} \to \mathcal{B}$ sending p_i to p_i. This is the same as an anti-linear anti-homomorphism from $\mathcal{A}$ to itself, hence the claim. □

Bibliography

[**BES94**] H. Bart, T. Ehrhardt and B. Silbermann. Zero sums of idempotents in Banach algebras. *Integral Equations and Operator Theory*, 19:125-134, 1994.