An Harris type result for the long-time behaviour of the Integrate and fire model

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Abstract

The Integrate and Fire model is a class of mean-field evolutionary equations which describes the activity of a population of neurons via their membrane potential. The structure of this equation is shared by many models of neural networks, and the investigation of its qualitative properties is still an open and challenging question. In this presentation we illustrate de basic mechanism of the I&F model and the idea behind Harris method [1]. Then, we apply this technique to the study of the asymptotic behaviour of the linear I&F model, which presents a singular boundary condition. We prove the exponentially fast convergence of its solutions to the unique stationary state in a L^1-weighted norm [3] and we support the result with some numerical simulations [2].

Keywords: mathematical neuroscience, Doeblin-Harris theorem, asymptotic behaviour

References

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